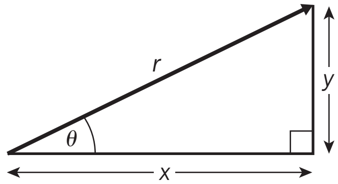
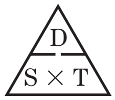
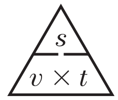
The 'If all else fails' list of Essential Formulae

Description	Formulae
Magnitude and direction of a vector given in component form	 $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$ $r^2 = x^2 + y^2$ or $r = \sqrt{(x^2 + y^2)}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$
Resolving a vector into perpendicular components	$x = r\cos\theta$ and $y = r\sin\theta$
Relationship between distance, speed and time	 or  where D = Distance, S = Speed, T = Time s = displacement, v = velocity, t = time
Average velocity	Average velocity $= \dfrac{\text{change in displacement}}{\text{change in time}}$
Average speed	Average speed $= \dfrac{\text{change in distance}}{\text{change in time}}$
The Uniform Acceleration Formulae (The SUVAT formulae)	$v = u + at$ $s = ut + \frac{1}{2}at^2$ $s = vt - \frac{1}{2}at^2$ $v^2 = u^2 + 2as$ $s = \frac{1}{2}(u + v)t$
Gravity and Weight	Acceleration due to gravity: $g = 9.8\ \text{ms}^{-2}$ Weight force: mg N
Newton's First Law	A body will continue to remain at rest or move at constant speed in a straight line unless an external force makes it act otherwise.
Newton's Second Law	A resultant force acting on a body produces an acceleration which is proportional to the resultant force. If we use SI units: $\mathbf{F} = m\mathbf{a}$
Newton's Third Law	For every action there is an equal and opposite reaction.
The definition of momentum	momentum $= mv$
The definition of impulse	$\mathbf{J} = \mathbf{F}t$
The Impulse–momentum principle	impulse = change in momentum or $J = mv - mu$
The Principle of Conservation of Linear Momentum (CLM)	total momentum before collision = total momentum after collision or $m_A u_A + m_B u_B = m_A v_A + m_B v_B$
Friction	$0 \leqslant F \leqslant F_{\text{max}}$, where $F_{\text{max}} = \mu R$
The moment of a force	$M = Fd$
The moment of a force at an angle	$M = Fd\sin\theta$,
Conditions for Equilibrium of a particle	For equilibrium, the Resultant force must be zero. i.e. $\boldsymbol{R} = \boldsymbol{0}$, where $\boldsymbol{0}$ is the zero vector or $\boldsymbol{R} = 0\boldsymbol{i} + 0\boldsymbol{j}$
Conditions for Equilibrium of a rigid body	The Resultant force in any direction must be zero ($\boldsymbol{R} = \boldsymbol{0}$) and The Sum of the Moments about any point must be zero ($M = 0$)

M1

Modular Maths for Edexcel

Jeff Trim

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All answers provided for examination questions are the sole responsibility of the author.

Introduction

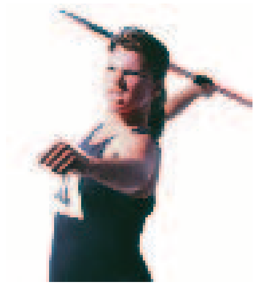
TO FOLLOW . . .

Contents

1 Basic Ideas

What is **mechanics**?
It is the mathematical study of physical situations.

Sometimes it is called **applied mathematics**.
This is because, like **statistics** and **discrete mathematics**, it is an example of how pure mathematics can be used to give information about ‘real-life’ situations.



The following three words are traditionally associated with mechanics: **kinematics**, **statics** and **dynamics**.
Why not research for yourself what these terms mean?
(You can try looking in a mathematical dictionary. However, they will be explained in the next section!)

In this chapter you will learn:
- about the modelling cycle,
- how to simplify real-life objects,
- the names for the most common models,
- how to choose the right model for a given problem,
- the meanings of typical modelling assumptions.

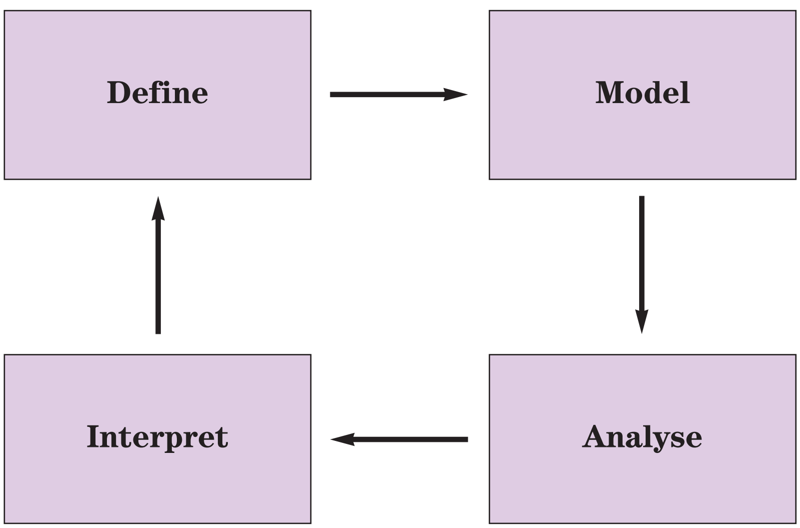
Modelling reality

Modelling is the name for the process of fitting mathematics to a real-life situation. It includes deciding what to put in and what to leave out.

The complexity of the model depends upon how accurate we need the answer to be. Simple models work best for simple problems.

This flow diagram is one way of describing the modelling process:

What do these words mean?
What is the significance of the arrows?



The four stages of the modelling process are explained like this:

Define
Consider the problem being asked and the answers required.
Decide what information is needed and what can be ignored.

Model
Simplify the problem down to its basics.
Choose the rules or formulae to use.
If necessary, estimate values for unknown quantities.
Make your assumptions clear.

Analyse
Using the rules chosen and the quantities given, work out the answer to the problem as you have modelled it.

Interpret
Review your answers in the light of the question.
Do they make sense? Are they accurate enough?
Are there other factors that need to be considered?

These four steps follow a natural sequence, indicated by the arrows.

Why is there an arrow from the last box back to the first?
After considering the results of our calculations, we may choose to revise our initial model. It may need to include new or different features.
This will lead to a refined model for a second attempt at the problem.

This is the 'modelling cycle' and applies equally well to statistics or discrete mathematics or any real-life use of mathematics.

Example 1

Suppose we need to calculate the distance travelled by a golf ball.

a) Can we ignore the size of the ball?
b) Is it safe to ignore air resistance?
c) What factors *must* we include in our model?

a) The size of the ball is so small compared to the distance it travels that we can safely ignore it.
b) Since air resistance is greater for larger surface areas and golf balls are relatively small, it can be ignored, *but* if a really accurate answer is needed it should be considered.
c) Essential factors are: the mass of the ball, the angle at which it is struck, its initial speed, gravity, the height of the ground where it is struck and where it lands, the time it is in the air.

TAKE IN A/W A1.1
TO BE SUPPLIED

Kinematics is the study of the motion of objects, looking at distance, speed, time and acceleration. (***Kinetic*** means *moving*.)

If the model includes the action of forces, then two other words are used.
Dynamics relates to moving objects. (***Dynamic*** means *changing*.)
Statics considers how forces balance to prevent movement.
(***Static*** means *still*.)

TAKE IN PHOTO P1.4
footballer (goalie)
TO BE SUPPLIED

Simplifying objects

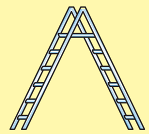
What is the main feature of everyday objects that we need to simplify? It is their shape that we ignore. Usually it is obvious how to replace the object with a simple mathematical model.

There are four main mathematical models that can be used. The one we choose will depend on the question being asked.

Common models

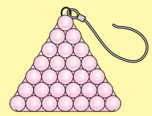
Particle
- all the mass acts at a single point with no volume or surface area
- air resistance is ignored in this model

Rigid body
- a simple shape with one or more parts where lengths are fixed
- it does not change shape



Rod
- a line whose length equals the object, but with no thickness
- it is a simple example of a rigid body

Lamina
- a flat plane, usually consisting of one or more common shapes
- its thickness is ignored
- it is another type of rigid body



The easiest model is the 'particle' or 'point mass'. This means we consider the object to be reduced to a single point, but still to have the same mass.

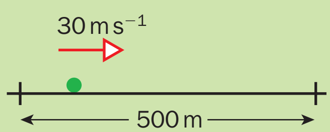
We only use a 'rigid-body' model if the distances or angles in the object are important to the solution of the problem.

Examples 2–5 show situations where each of these models might be used. As you look at the diagrams, try to think of some more examples of your own.

Example 2

A car is driven for 500 metres along a straight section of motorway at $30\,\text{m}\,\text{s}^{-1}$.

As it is travelling a fair distance, the car can be modelled as a particle.



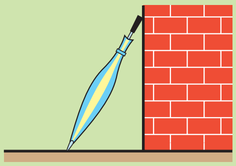
The important thing to remember is that the model chosen depends also on the question being asked. In the example above, the length of the car is unimportant because it is far less than the distance it travels.

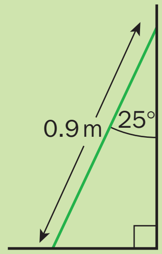
If the question were about how a car accelerates and decelerates as it travels over speed bumps placed 5 metres apart, the length of the car would matter!

Example 3

An umbrella of length 0.9 m is leaning against a wall so that it makes an approximate angle of 25° with the wall.

As the umbrella is basically a straight object, it can be considered to be a rod.



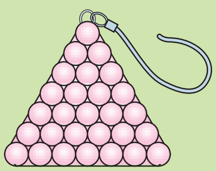


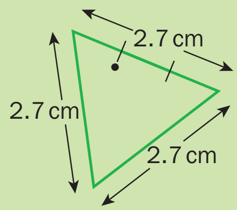
Does it matter if the handle is heavier than the rest of the umbrella?
We usually consider a rod to be 'uniform'. This means that the mass is distributed equally throughout its length.
We are ignoring the fact that the handle may be particularly heavy compared to the rest of the umbrella.

Example 4

A flat earring in the shape of an equilateral triangle of side 2.7 cm is suspended from a point two-thirds of the way along one side.

The shape and lengths are important to this question, but as the earring is flat and presumably quite thin, a lamina is the best model.



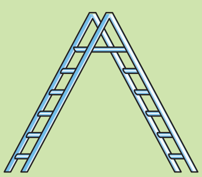


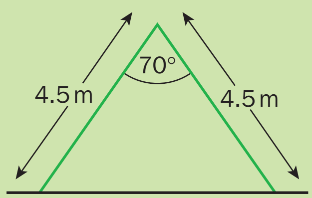
Like a uniform rod, we consider a uniform lamina to have its mass spread out equally throughout its area.

Example 5

A folding step ladder of total length 9 m is opened out so that the angle between the sides is 70°.

This object needs more than just a single rod to represent it. A rigid body made up of two uniform rods will be the best model.





How would this last example change if there were a bracing strut between the two sides?
This would become a 'framework'. This is the name for structures made up of connected rods. This is quite a sophisticated model and is not required for AS level mechanics.

Assumptions

When we *define* the model for a particular situation, we also consider any **assumptions** we will make. These are the simplifications we permit.

TAKE IN PHOTO P1.7
toy car on slope
TO BE SUPPLIED

As we begin the study of mechanics it is best to choose the simplest possible model, but recognise that this limits the accuracy of our calculations. Later on, as we add more complicated modelling ideas, we can use increasingly sophisticated models to achieve more precise results.

Suppose a toy car is rolling down a slope.
What features of the real-life situation could we ignore?
We would probably decide that the effect of air resistance would be too small to include in our model. If this were outdoors, we would ignore the wind speed and direction too. We might also assume the slope has no dips or bumps and that the car travels in a perfectly straight line.
We will definitely assume that gravity makes the car go down the hill.

Is gravity always the same?
In fact there are variations depending on where you are on the Earth's surface and how far you are above or below sea level.
In general we stick to one value for the acceleration due to gravity ($g = 9.8\,\mathrm{m\,s^{-2}}$) and assume that it will apply throughout the problem.
We will look at gravity in more detail later, in chapter 5.

You need to learn the following words that regularly feature in exercises and examination questions to indicate the assumptions being made.

Typical assumptions

Light	• having no mass
Inextensible/inelastic	• does not stretch (e.g. for a connecting cable)
Smooth	• there is no friction
Rough	• friction is acting
No air resistance	• this force is ignored (e.g. for a particle)

The assumptions we make allow us to solve the problem more easily. However, when we come to interpret our answers, it is a good idea to consider how these assumptions might have affected the answer.

TAKE IN PHOTO P1.8
netball scene
TO BE SUPPLIED

Suppose we are calculating the distance that a netball travels if thrown through the air. Imagine our model ignored air resistance and produced an answer of 20 m.

How might air resistance have affected the outcome?
Air resistance would have acted against the motion of the netball and so reduced the distance of 20 m a little.
It is not necessary to guess by how much the distance might have changed, but it is important to have some idea of the possible effect.

The netball might also be spinning. What difference might that make?

Summary

- **Mechanics** is the study of modelling real-life situations in terms of mathematics.
  It can be broken into this four-step cycle: **Define** → **Model** → **Analyse** → **Interpret** (→ define).
- Three important aspects of mechanics are:
  Kinematics – acceleration, speed, time and distance
  Dynamics – how forces produce or change motion
  Statics – how forces balance to prevent motion.
- The most common model for an object is the **particle**, which has no surface area and therefore we can ignore air resistance.
- If the lengths or the shape are important we use the **rod**, **lamina** or **rigid body**.
- **Assumptions** are made to keep the mathematical model simple by ignoring certain factors.
- The nature of the question helps to determine the appropriate model and the assumptions to be made.

End of Chapter Questions

1 Can you complete these sentences?

a) Mechanics is the study of …

b) The modelling cycle includes these four stages: …

c) Three words classically associated with mechanics are: …

d) A 'point mass' is another name for a …

e) If lengths are important, the model used is a …

f) Air resistance is ignored in the … model.

g) A cable does not stretch if it is inextensible or …

h) Simplifications are also called …

2 What word describes the assumption that an object's mass is small enough to be ignored?

3 Which model is always used for a flat object?

4 Why is it reasonable to ignore air resistance on a particle?

5 Make a list of at least six features that would be ignored by modelling a car as a particle.

6 Match up each of these words {Kinematics, Statics, Dynamics} with its meaning from this list {changing, still, moving}.

7 Think of a real-life situation where air resistance would *not* be ignored.

8 Which model would you use for a plank resting against a wall?

9 What information would you collect to predict how long a textbook would take to fall to the ground from an upper-storey classroom window?

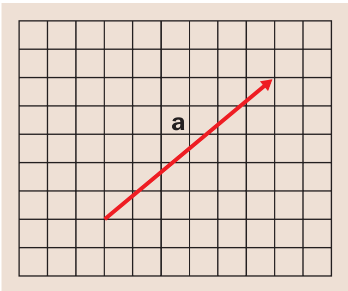
10 What information do you need in order to study how a barrel rolls down a sloping road?

How to make the Examiner happy

- Make sure you use the proper name for the model that is being used: particle, rod, lamina or rigid body
- Be prepared to state the modelling assumptions that apply to a specific model.
- Exam questions often include a one-liner such as "Which real-life features are ignored if a particle model is used?" The answer will probably be "air resistance" and maybe also "rotation of the object" and "the length of the object"

2 Using Vectors

Can you remember what a **vector** is?
You will have used them in the past to describe a movement on a graph.



across 6, up 5

This vector is written: $\mathbf{a} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$

In mechanics, vectors have a special use to help us solve problems in two or three dimensions.

In this chapter you will learn:

- how to write down and use vectors,
- how to find the magnitude and direction of a vector,
- how to resolve a vector into perpendicular components,
- how to add, subtract and find a multiple of a vector,
- how to use vectors to represent different quantities in mechanics.

Scalars and vectors

A **scalar** quantity is something which has size (magnitude) but no particular direction. It is just a single number.

A **vector** quantity has size and direction. It needs two pieces of information to describe it fully.

Do **speed** and **velocity** mean the same thing?
No. In mechanics there is a strict difference between them.

- Speed is *how fast you are going*.
  For example, 'a car is travelling at 25 m s^{-1}'.
- Velocity is *how fast you are going and in which direction*.
  For example, 'a car is travelling at 25 m s^{-1}, due North'.

Out of 'speed' and 'velocity', which is the scalar and which is the vector?
Speed is the scalar. It is just one piece of information.
Velocity is the vector. It requires two pieces of information.

Example 1

Is 'time' a scalar quantity, or is it a vector quantity?

'Time' is a scalar quantity, as it is not associated with a particular direction.

Writing vectors

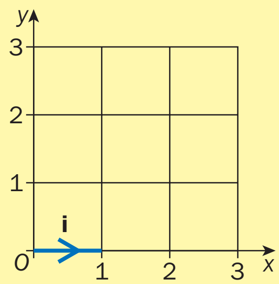
There are several different ways of writing vectors.
Which ones have you met before?

Look back to the vector $\begin{pmatrix} 6 \\ 5 \end{pmatrix}$. This is an example of a **column vector**.
We can write the same vector using 'unit vectors'.

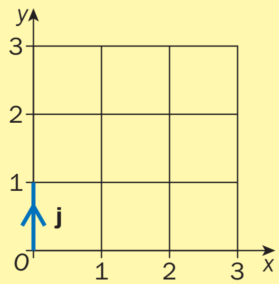
Vector Notation
Although column vectors are used in this chapter to introduce the concept of a vector, examination questions will usually present vectors in unit vector form.

The unit vectors:

$\mathbf{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$



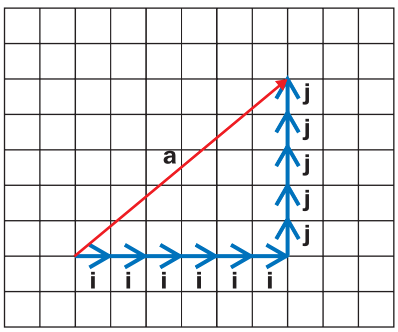
$\mathbf{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



i is 'one step across'

j is 'one step upwards'

They are called 'unit vectors' because each has a length of one. (Unity = one.)



We can write: $\mathbf{a} = 6\mathbf{i} + 5\mathbf{j}$

Did you notice that the letters representing the vectors have been written in **bold type**?
This is to show clearly which letters stand for vectors, and which for ordinary numbers (i.e. scalars). Textbooks and exam papers use this convention.

It is not practical to do this when writing vectors by hand, so the convention for written work is to **underline** the vectors.

Writing vectors by hand:
Always underline the letters that stand for vectors.

For example, if you see: $\mathbf{a} = 6\mathbf{i} + 5\mathbf{j}$, you must write: $\underline{a} = 6\underline{i} + 5\underline{j}$

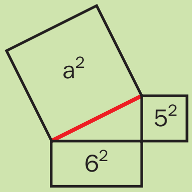
Magnitude and direction

How can we find the length (the magnitude) of a vector such as $\begin{pmatrix}6\\5\end{pmatrix}$?

Since it can be drawn using a right-angled triangle, we can use Pythagoras' Theorem to find the longest side.

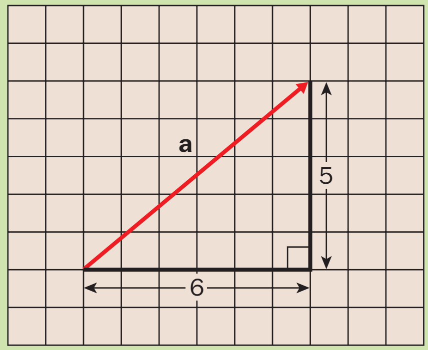
Pythagoras' Theorem
In any right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

Example 2
What is the magnitude of the vector $\begin{pmatrix}6\\5\end{pmatrix}$?



By Pythagoras' Theorem:

$$\begin{aligned}\text{Magnitude of } \mathbf{a} &= \sqrt{(6^2 + 5^2)}\\ &= \sqrt{(36 + 25)}\\ &= \sqrt{61}\\ &= 7.81 \text{ (3 significant figures or s.f.)}\end{aligned}$$



This is now a scalar and not a vector,

so we write: $a = \underline{7.81}$ (3 s.f.)

N.B. 'Magnitude of **a**' is also written $|\mathbf{a}|$.

What about the angle the vector **a** makes with the horizontal direction?
As it is a right-angled triangle, we can use trigonometry.

Example 3
What angle does the vector $\begin{pmatrix}6\\5\end{pmatrix}$ make with the horizontal direction?

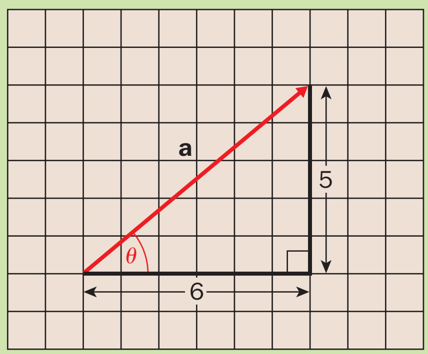
Using trigonometry:

$$\text{opp} = 5,\ \text{adj} = 6 \ \text{ and } \ \tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan\theta = \tfrac{5}{6}$$

$$\theta = \tan^{-1}\left(\frac{5}{6}\right)$$

$$\theta = \underline{39.8°} \text{ (1 decimal place or 1 d.p.)}$$



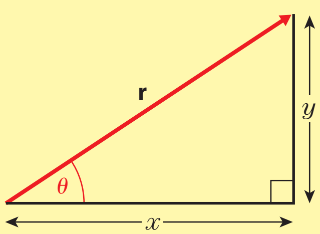
Finding the magnitude and direction of a vector given in component form:

$$r^2 = x^2 + y^2 \quad \text{or} \quad r = \sqrt{(x^2 + y^2)}$$

and

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

As r is a length, $r > 0$, so ignore the negative square root.



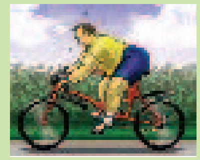
$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$

NB: r means 'the magnitude of **r**' or $|\mathbf{r}| = r$

Sometimes the unit vectors are assigned to specific directions, such as East and North, in order to provide the context for a question.

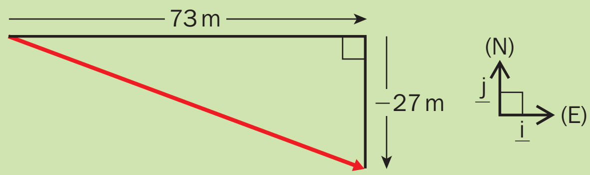
Example 4

After a short journey, the displacement of a cyclist is $(73\mathbf{i} - 27\mathbf{j})$ m, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in directions due East and due North respectively. Assuming the journey is in a straight line, what is the total distance the cyclist has travelled?

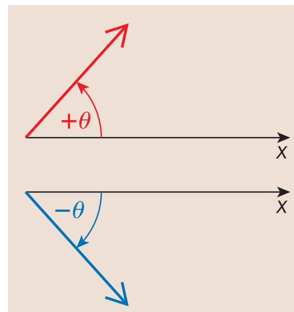


Even though there is a negative $\mathbf{j}$ component, use Pythagoras' Theorem, as before:

$$\begin{aligned}\text{Distance} &= \sqrt{(73^2 + (-27)^2)} \\ &= \sqrt{(5329 + 729)} \\ &= \sqrt{6058} \\ &= \underline{77.8 \text{ m}} \text{ (3 s.f.)}\end{aligned}$$



Did you understand the difference between **displacement** and **distance** in the last example?
Displacement refers to the vector, but distance just means the length of this vector.



When you are finding the direction of a vector with negative components, it is worth drawing a sketch first, to judge if the answer makes sense.

It is a mathematical convention to measure angles **anticlockwise** from the positive x direction.
If a **clockwise** angle is used, it is given a *negative* sign.

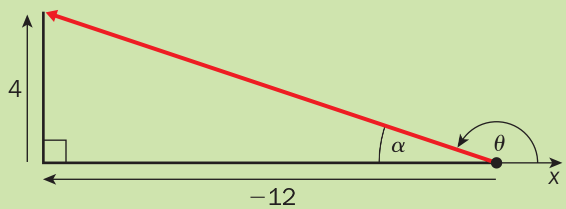
Example 5

Find the angle between the x direction (i.e. horizontal, to the right) and the vector $\begin{pmatrix} -12 \\ 4 \end{pmatrix}$.

Using the normal rule: $\quad \alpha = \tan^{-1}\left(\dfrac{y}{x}\right)$

$$= \tan^{-1}\left(\frac{4}{-12}\right)$$

$$= -18.4° \text{ (1 d.p.)}$$



Looking at the diagram, it is clear that we need an angle of
$\theta = 180° + \alpha = 180° + {}^{-}18.4° = \underline{161.6°}$ (1 d.p.)

If you research the tangent function in a Pure Maths textbook, you can find out why it might be necessary to add on 180° to the first answer. However, as long as you always draw a sketch, there should be no confusion.

Why were the angles given correct to 1 decimal place, and not 3 significant figures?
In example 5, we know that the two angles mentioned should add up to 180°. If the final answer were given as 162°, they would appear to add up to 180.4°, which might be confusing.

Resolving into components

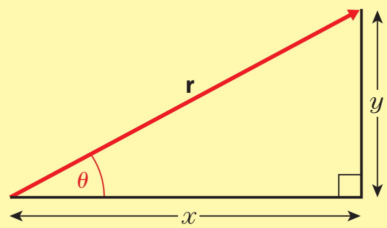
We can also use trigonometry to convert a vector from a length in a given direction to component form.
Once we have the horizontal and vertical components, we can write the vector as a column vector or using **i**'s and **j**'s.

Resolving a vector into perpendicular components:

$$x = r\cos\theta$$

and

$$y = r\sin\theta$$



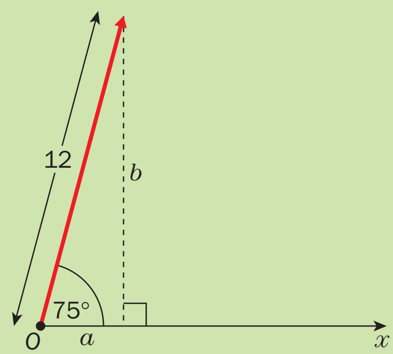
We will always be using **sine** to find the side **opposite** the angle and **cosine** to find the side **next to (adjacent to)** the angle.

Example 6

Express in the form $a\mathbf{i} + b\mathbf{j}$, a vector of length 12 units at an angle of 75° to the x direction.

It is always a good idea to begin with a sketch of the vector.

Label the required sides.



$$a = 12 \times \cos 75°$$
$$= 3.11 \text{ units (3 s.f.)}$$
$$b = 12 \times \sin 75°$$
$$= 11.6 \text{ units (3 s.f.)}$$

$\therefore$ vector $= (3.11\mathbf{i} + 11.6\mathbf{j})$ units

Do you remember the three rules for writing down a direction as a bearing?
Bearings are always:

- measured clockwise,
- from north,
- given using three digits.

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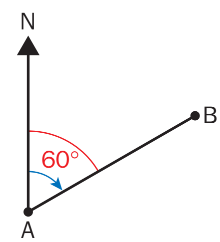
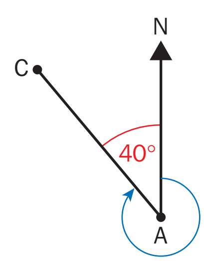


The bearing of B from A is 060°.

How does this differ from the convention for measuring angles from the x direction?
With bearings, **clockwise** is the chosen direction, but usually we take an **anticlockwise** angle as being positive.

Mechanics questions are often written in terms of bearings.
It is always best to draw a sketch before beginning the solution.

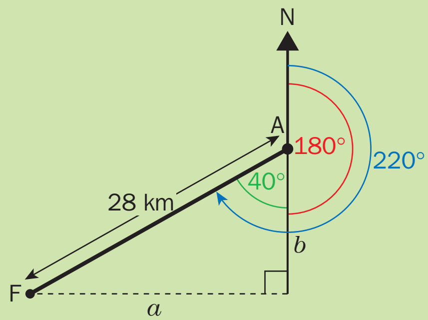


The bearing of C from A is 360° − 40° = 320°.

Example 7

A farm (F) lies at a distance of 28 km, on a bearing of 220° from an airfield (A). Convert this displacement into a column vector, assuming East and North are the x and y directions.

Once again, sketch the situation first.



We need to think of the airfield as the 'origin' for the displacement. It will be easiest to use the angle of 40° in our calculations.

$$\begin{aligned} a &= 28 \times \sin 40° \\ &= 18.0 \text{ km (3 s.f.)} \\ b &= 28 \times \cos 40° \\ &= 21.4 \text{ km (3 s.f.)} \end{aligned}$$

Now we need to make sure that the components have negative signs, for 'left' and 'down':

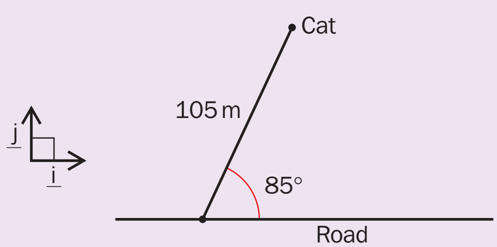
$\therefore$ displacement $= \underline{\begin{pmatrix} -18.0 \\ -21.4 \end{pmatrix} \text{km}}$

Practice

1 For the displacement vector $\mathbf{s} = (-11\mathbf{i} + 29\mathbf{j})$ m, find the magnitude of $\mathbf{s}$ and the angle it makes with the $\mathbf{i}$ direction.

2 What is the magnitude of the column vector $\begin{pmatrix} 8 \\ 15 \end{pmatrix}$?

3 A cat scampers 105 m in a straight line at an angle of 85° to a straight road. Taking $\mathbf{i}$ and $\mathbf{j}$ to act in the direction shown in the diagram, resolve this displacement into the form $a\mathbf{i} + b\mathbf{j}$.



4 A ship is visible on a bearing of 195° from a lighthouse. Its distance is estimated to be 6 km. Resolve this displacement into component form, where $\mathbf{i}$ is a unit vector due East and $\mathbf{j}$ is a unit vector due North.

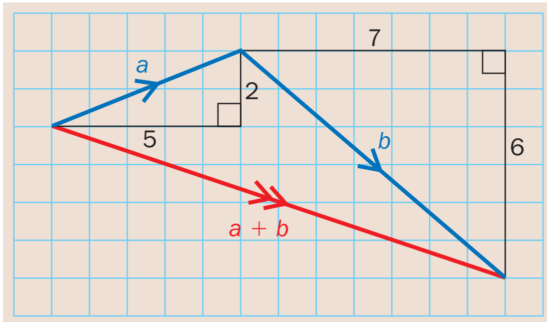
Adding vectors

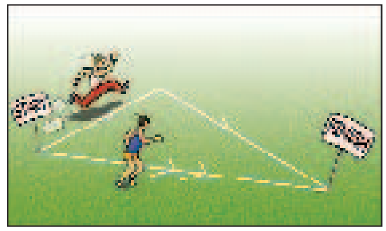
Given two vectors, $\mathbf{a} = 5\mathbf{i} + 2\mathbf{j}$ and $\mathbf{b} = 7\mathbf{i} - 6\mathbf{j}$,
how can we add them and what does the result mean?

Your first reaction is probably to treat the separate components of the vectors like algebraic quantities and to add like terms.
This is completely correct!

So: $$\begin{aligned}\mathbf{a} + \mathbf{b} &= (5\mathbf{i} + 2\mathbf{j}) + (7\mathbf{i} - 6\mathbf{j})\\ &= 12\mathbf{i} - 4\mathbf{j}\end{aligned}$$

The meaning of this result is best seen from a diagram.





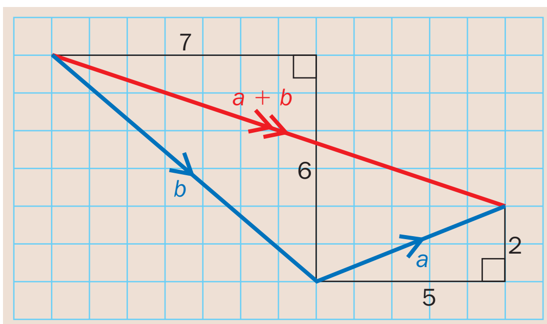
$\mathbf{a} + \mathbf{b}$ is the single journey which is the short cut from the start of $\mathbf{a}$ to the end of $\mathbf{b}$, when $\mathbf{a}$ is followed by $\mathbf{b}$.

What happens if we have $\mathbf{b}$ followed by $\mathbf{a}$?
By adding we get:

$$\begin{aligned}\mathbf{b} + \mathbf{a} &= (7\mathbf{i} - 6\mathbf{j}) + (5\mathbf{i} + 2\mathbf{j})\\ &= 12\mathbf{i} - 4\mathbf{j}\end{aligned}$$

Is it surprising that we get the same answer?
Normal addition is **commutative**, meaning that you get the same answer in whichever order you add the numbers.

Let's look at a diagram.



We can see for this case that the shortcut is the same vector, no matter the order in which we travel along the two vectors.

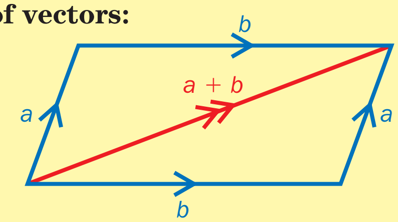
The **sum** of two or more vectors is called the **resultant** vector.

The parallelogram rule for addition of vectors:

Consider two general vectors, **a** and **b**:

$$\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$$

This shows that addition of vectors is commutative.



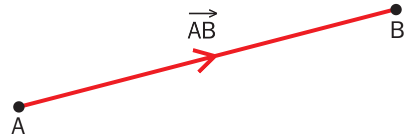
Example 8

If $\mathbf{p} = 4\mathbf{i} + 12\mathbf{j}$, $\mathbf{q} = -7\mathbf{i} - 2\mathbf{j}$ and $\mathbf{r} = 5\mathbf{i} - 8\mathbf{j}$, find $\mathbf{p} + \mathbf{q} + \mathbf{r}$.

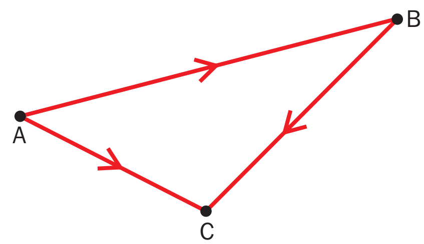
Simply add together all the **i** terms and all the **j** terms.

$$\begin{aligned} \mathbf{p} + \mathbf{q} + \mathbf{r} &= (4\mathbf{i} + 12\mathbf{j}) + (-7\mathbf{i} - 2\mathbf{j}) + (5\mathbf{i} - 8\mathbf{j}) \\ &= (4\mathbf{i} - 7\mathbf{i} + 5\mathbf{i}) + (12\mathbf{j} - 2\mathbf{j} - 8\mathbf{j}) \\ &= \underline{2\mathbf{i} + 2\mathbf{j}} \end{aligned}$$

Sometimes a vector is referred to by the letters at either end.
$\overrightarrow{AB}$ is the vector from A to B.



In this diagram the vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{AC}$ are shown.



Using the principle of the resultant being the shortcut:

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

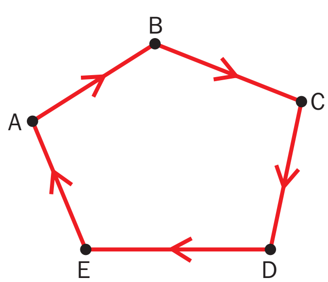
Do you notice how, like dominoes, the inside letters of the vector sum match up? The answer is then the outside pair of letters.

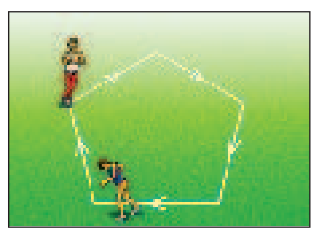
What will be the answer to $\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RS} + \overrightarrow{ST}$?

It will be $\overrightarrow{PT}$.

Look at the next diagram and consider this question.

What will you get if you add: $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA}$?





Is it $\overrightarrow{AA}$? Or 0?
When a vector sum equals zero, we must write something to indicate that it is the **zero vector**. We show this by $\mathbf{0}$ or $\underline{0}$ or $0\mathbf{i} + 0\mathbf{j}$ or $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

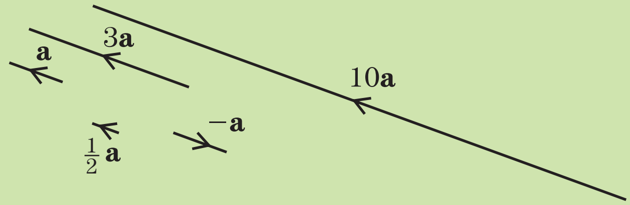
Parallel vectors

What about $3\mathbf{a}$, or $10\mathbf{a}$, or $\frac{1}{2}\mathbf{a}$ or $-\mathbf{a}$?
If we follow the rules of ordinary algebra and multiply the separate coefficients of $\mathbf{i}$ and $\mathbf{j}$ by the new number, what will that look like on a diagram?

Example 9

If $\mathbf{a}$ is the vector $-2\mathbf{i} + \mathbf{j}$, write down and sketch the vectors $3\mathbf{a}$, $10\mathbf{a}$, $\frac{1}{2}\mathbf{a}$ and $-\mathbf{a}$.

$$3\mathbf{a} = 3 \times (-2\mathbf{i} + \mathbf{j}) = \underline{-6\mathbf{i} + 3\mathbf{j}}$$
$$10\mathbf{a} = 10 \times (-2\mathbf{i} + \mathbf{j}) = \underline{-20\mathbf{i} + 10\mathbf{j}}$$
$$\tfrac{1}{2}\mathbf{a} = \tfrac{1}{2} \times (-2\mathbf{i} + \mathbf{j}) = \underline{-\mathbf{i} + \tfrac{1}{2}\mathbf{j}}$$
$$-\mathbf{a} = -(-2\mathbf{i} + \mathbf{j}) = \underline{2\mathbf{i} - \mathbf{j}}$$



Can you work out the effect of multiplying the vectors by these numbers?
The new vector is parallel to the original vector and the number is the length scale factor.
The minus sign in the last example caused the vector to reverse direction.

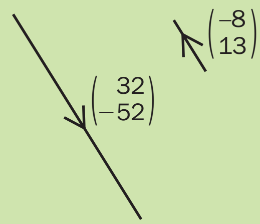
The same principles work for vectors written in column form.

Example 10

Write down the vector that is four times as long and in the opposite direction to $\begin{pmatrix} -8 \\ 13 \end{pmatrix}$.

The required vector is: $-4 \times \begin{pmatrix} -8 \\ 13 \end{pmatrix} = \begin{pmatrix} (-4) \times (-8) \\ (-4) \times 13 \end{pmatrix}$

$$= \underline{\begin{pmatrix} 32 \\ -52 \end{pmatrix}}$$



Modelling with vectors

As we study the mathematics of motion (**kinematics**) and of equilibrium (**statics**), we will use these letters to represent common quantities:

In two or more dimensions:		In one dimension or magnitude only:	
$\mathbf{s}$	displacement	x	distance
$\mathbf{u}$	initial velocity	u	initial speed
$\mathbf{v}$	final velocity	v	final speed
$\mathbf{a}$	acceleration	a	acceleration

Arrow notation
The following convention for the shapes of arrows is used throughout this book:

	force		resultant force
	velocity		resultant velocity
	displacement		resultant displacement
	acceleration		distance, dimensions

Maths in Action: How to Get Away with a Forward Pass

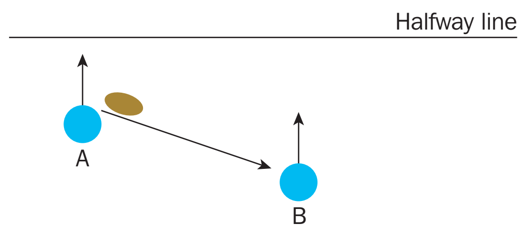
One of the most important rules in rugby states that a player is only able to pass the ball to somebody who is behind him. Without this rule, rugby would be a much less orderly game, with players scattered across the field, but instead when players are putting together a string of passes they usually form a neat line across the field.

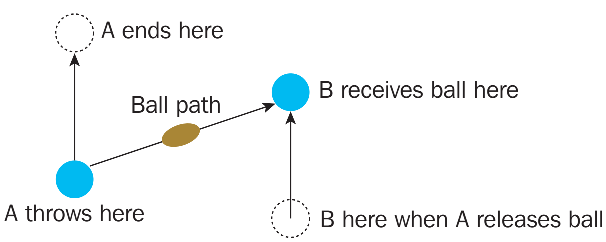
PICTURE TO BE INSERTED
TO BE SUPPLIED

A forward pass is penalised and the referee calls a scrum. The odd thing, however, is that if you were watching a rugby game from a helicopter hovering above the pitch, you would see "forward passes" going *un-penalised* all the time.

Vectors can explain why this happens.

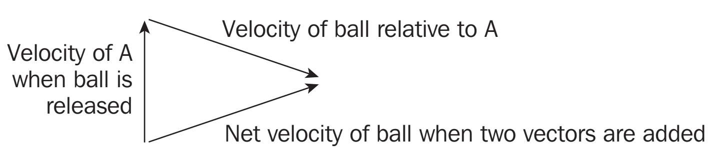
In the diagram below, player A has the ball, and wants to pass it to player B. Both players are running at full speed towards the halfway line across the pitch. Viewed from our helicopter, we can clearly see that player B is further back than player A, so the direction in which A points the ball is indeed backwards, as the rugby rule states.





However, consider what happens to player B while the ball is floating in the air towards him. He will travel a couple of metres forwards, and for the pass to be successful, it needs to land in his hands, not two metres behind him. This means that the actual path of the ball must look like this:

From our helicopter we can now clearly see that the ball has gone forward! So how can A have thrown the ball backwards yet it went forwards? The reason is that the ball DID go backwards *relative to A*. We can draw a triangle of velocities to show the complete picture:



To a television viewer, this forward motion of the ball is often quite obvious, especially if the pass happens close to a line on the pitch. But as long as player A keeps running forward, the referee is unlikely to blow his whistle.

Summary

- **Scalars** have **magnitude** only, e.g. time, mass, speed
- **Vectors** have **magnitude** and **direction**, e.g. displacement, velocity, acceleration
- We can find the magnitude of a vector by using **Pythagoras' Theorem**
- We can find the direction angle of a vector by using **trigonometry** (**tangent**)
- We can resolve vectors into **components** using **sine** and **cosine**
- Scalar multiples of vectors are **parallel**

End of Chapter Questions

1 Can you complete these sentences?

a) A vector quantity has magnitude and …

b) A scalar quantity only has …

c) Books show vectors by using … type.

d) Show vectors in written work by …

e) Find the magnitude of a vector by using …

f) Find the direction of a vector by using …

g) Resolve a vector into components by using … and …

h) Scalar multiples of a vector are … to each other.

2 Is 'density' a scalar or a vector quantity?

3 Find the magnitude of the vector $24\mathbf{i} - 10\mathbf{j}$.

4 What angle does the vector $\begin{pmatrix} -17 \\ -8 \end{pmatrix}$ make with the x direction?

5 A vector has length 5.8 units and is at an angle of 150° from the positive x direction. Express this vector in the form $a\mathbf{i} + b\mathbf{j}$.

6 A displacement vector is 300 m long, with a bearing of 312°. Write this as a column vector taking **i** as a unit vector due East and **j** as a unit vector due North.

7 $\mathbf{a} = \begin{pmatrix} 12 \\ -7 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$. Write down $\mathbf{a} + \mathbf{b}$ and show these three vectors on a sketch.

8 A, B, C and D are the four corners of a square. What is $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA}$?

9 Which of these vectors are parallel:

$\begin{pmatrix} 5 \\ -1 \end{pmatrix}, \begin{pmatrix} 10 \\ 50 \end{pmatrix}, \begin{pmatrix} -25 \\ 5 \end{pmatrix}, \begin{pmatrix} 2.5 \\ -0.5 \end{pmatrix}, \begin{pmatrix} 30 \\ -6 \end{pmatrix}$?

10 Write down the vector which is half as long and in the opposite direction to $-18\mathbf{i} + 12\mathbf{j}$.

11 Given that $\mathbf{u} = t\mathbf{i} + 8\mathbf{j}$ and that the magnitude of u is 17, find the two possible values of t.

12 Find, in degrees to 1 d.p., the acute angle made with the vector **j** by each of the following vectors:

a) $\mathbf{i} + \mathbf{j}$ b) $4\mathbf{i} + 6\mathbf{j}$

c) $-\mathbf{i} + 9\mathbf{j}$ d) $3\mathbf{i} + 7\mathbf{j}$

13 Find, in the form a**i** + b**j**,

a) a vector of magnitude 30 in the direction of the vector $3\mathbf{i} - 4\mathbf{j}$,

b) a vector of magnitude 5 in the direction of the vector $7\mathbf{i} - 24\mathbf{j}$.

14 Find a unit vector (a vector of length 1 unit) in the direction

exam

a) $6\mathbf{i} + 8\mathbf{j}$　　b) $7\mathbf{i} - 24\mathbf{j}$

c) $\mathbf{i} - \mathbf{j}$　　d) $-2\mathbf{i} + 6\mathbf{j}$

15 Given that the vector $3\mathbf{i} + b\mathbf{j}$ is parallel to $12\mathbf{i} - 4\mathbf{j}$, find the value of b.

exam

16 Given that $\mathbf{p} = \mathbf{i} - 3\mathbf{j}$ and $\mathbf{q} = 2\mathbf{i} + \mathbf{j}$, find the values of a and b for which

exam

a) $\mathbf{p} + a\mathbf{q}$ is parallel to the vector $\mathbf{i}$,

b) $b\mathbf{p} + \mathbf{q}$ is parallel to the vector $\mathbf{j}$.

17 Given that the vector $(2t - 5)\mathbf{i} - (4 - t)\mathbf{j}$ is parallel to the vector $\mathbf{i} - \mathbf{j}$, find the value of t.

exam

18 Two particles A and B are moving in a plane. The velocity of A at time t seconds is given by $\mathbf{v}_A = (t - 2)\mathbf{i} + (2t - 3)\mathbf{j}\,\text{ms}^{-1}$ and the velocity of B at time t seconds is given by $\mathbf{v}_B = \mathbf{i} + t\mathbf{j}\,\text{ms}^{-1}$. Find the values of t when the two particles are moving in the same direction.

exam

19 Taking $\mathbf{i}$ and $\mathbf{j}$ as unit vectors due east and due north respectively,

exam

a) find, in the form $a\mathbf{i} + b\mathbf{j}$, the resultant of a vector of magnitude 10 on a bearing of 060° and a vector of magnitude 30 on a bearing of 210°.

b) Hence find the magnitude and direction of the resultant.

How to make the Examiner happy

- Always underline all the vectors, especially all those i's and j's
- Draw a right-angled triangle diagram when resolving vectors into components. You don't have to draw it to scale, but if it is reasonably accurate, you can tell if your answers make reasonable sense

3 Motion in a Straight Line

In order to build up a model for **motion** and to develop some procedures to analyse it, we will start with movement that takes place only in a straight line. Some examples could be: dropping a ball vertically, driving a car on a straight road or rolling a snooker ball along a flat surface.
Can you think of some more examples?

TAKE IN PHOTO P3.1
snooker ball being hit
TO BE SUPPLIED

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The first thing to do is to describe the motion precisely and perhaps draw a suitable diagram. We will also be using graphs to record any changes in the motion and to calculate additional information.

In this chapter you will learn:
- how to calculate average speed and average velocity,
- the meaning of acceleration, deceleration and retardation,
- how to interpret different types of graphs,
- how to use the gradient of a line on a graph,
- how to use the area under a section of a graph,
- how to use units to determine the relevance of the area or gradient.

Displacement, velocity and acceleration

Do you know the difference between 'distance' and 'displacement'?
In Chapter 2 we learnt that a scalar quantity has magnitude (or size), but a vector quantity has magnitude *and* direction.
Distance is a scalar, but displacement is a vector.

Putting it simply:
- **distance** means 'how far'
- **displacement** means 'how far and in what direction'.

Example 1

Describe the position of Reading relative to London.

TAKE IN PHOTO P3.3
(map of towns in Berkshire and London)
TO BE SUPPLIED

Reading is approximately 60 km due west of London.
This is the *displacement* of Reading from London, as we know the direction.
We could also say that the *distance* to Reading from London is 60 km.

There is a similar distinction between 'speed' and 'velocity'.
Do you know which is which?
Speed is a scalar and velocity is a vector.
Speed is the magnitude of the velocity vector.

Putting it simply:

- **speed** means ‘how fast’
- **velocity** means ‘how fast and in what direction’.

Speed tells us the rate at which an object travels through a distance.

Speed and velocity:

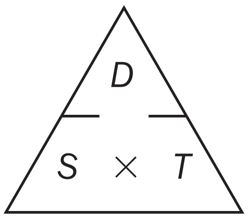
Speed is the rate of change of distance with respect to time. (scalar)

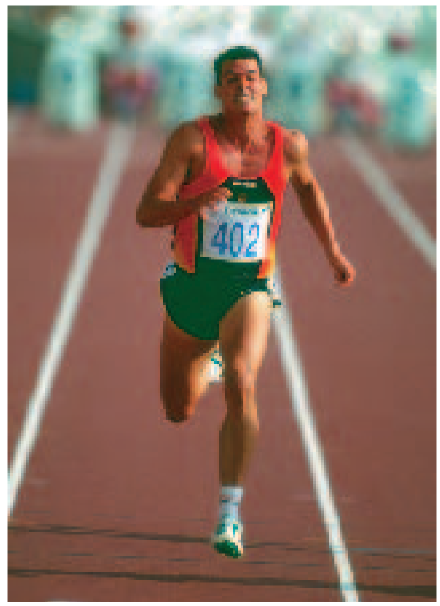
Velocity is the rate of change of displacement with respect to time. (vector)

Rules or relationships that are true for velocity and displacement will usually be true for speed and distance as well. To avoid repetition, this text will most often refer to the vector forms rather than the scalar forms.

Distance, Speed and Time
This triangle (below left) will be familiar to you from Science or Physics. Look out, however, for the letters or variables that are used to stand for distance, speed and time in Mechanics.
(See p. 32)

You probably already know the connection between distance, speed and time. It can be summarised in the following triangle diagram:





By covering up the variable required, the relationship between the other two is seen.

So, we have: $D = S \times T \quad or \quad S = \dfrac{D}{T} \quad or \quad T = \dfrac{D}{S}$

These rules are true for constant speed. However, if it is changing we need to consider the rate at which the speed is increasing or decreasing.
This is called the **acceleration**, and will have a positive value if the speed is increasing or a negative value if the speed is decreasing.

Is acceleration a scalar or a vector?
In fact the same word is used in both cases, so we need to decide how to interpret it from the context of a question.

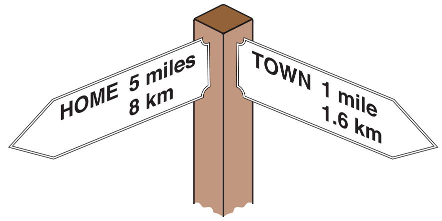
When an object is ‘decelerating’ or ‘retarding’, this means its speed is decreasing and therefore the *acceleration* is negative.
Deceleration and retardation are usually given as positive quantities.

Acceleration and deceleration

Acceleration is the rate of change of velocity (or speed) with respect to time.

Deceleration (or **retardation**) is the magnitude of a negative acceleration.

Distance–time graphs

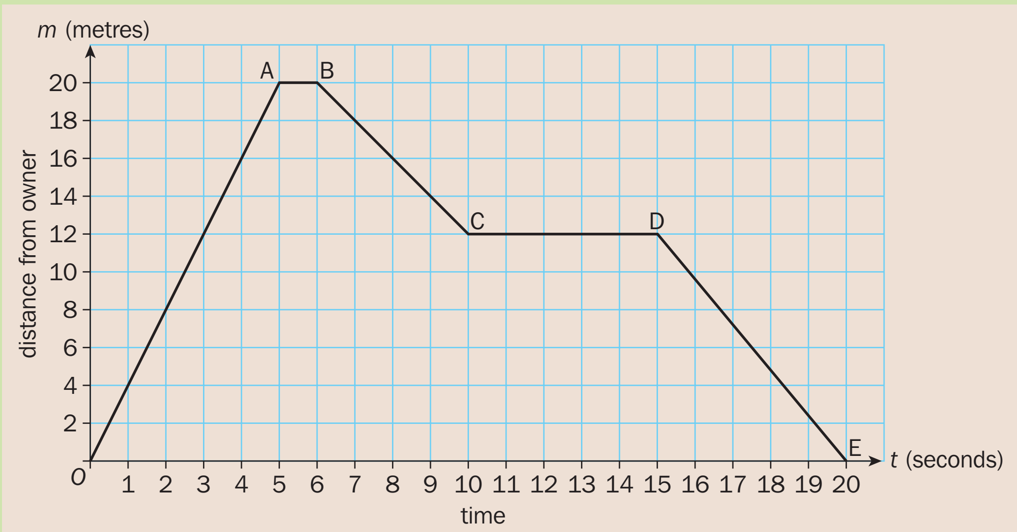


You will have met 'travel graphs' before.
How are the axes normally labelled?
They are usually graphs of distance plotted against time.
Time is always shown on the horizontal axis and the vertical axis is labelled something like 'distance from home' or 'distance from London'.

Graphs like these are also referred to as distance–time graphs.

Example 2

The graph below shows the distance of a dog from its owner after a stick is thrown for it to fetch.
First it runs to the stick, pauses briefly to pick up the stick and then returns to its owner.
Its return journey is interrupted when it decides to sniff an interesting scent on the grass.



a) How far did the dog run to pick up the stick?
b) How long did it delay during its return trip to sniff the scent?
c) Can we calculate the speed of its outward journey?

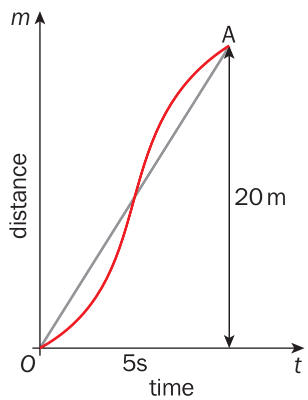
a) Section OA shows the outward journey. The dog runs 20 m.
b) The horizontal line CD shows where the dog stopped its return journey. This lasted 5 s.
c) Section OA shows a distance of 20 m travelled in 5 s.

$$\begin{aligned} \text{speed} &= \text{distance} \div \text{time} \\ &= 20 \div 5 \\ &= \underline{4\ \text{m}\,\text{s}^{-1}} \end{aligned}$$

Is there any other possible interpretation of the section of the graph labelled CD, where the dog is sniffing the scent?
It is at a constant distance of 12 m from the owner, but could possibly be moving in an arc of a circle rather than remaining stationary.
As this is only a graph of distance rather than displacement, we can't tell, as we don't know if the direction from the dog to its owner is changing.

What mathematical assumption are we making by supposing that the sloping lines on the graph are straight?
The speed of the dog would need to be constant for the line to be straight.
This is clearly a simplification of what might really have happened.
What do you think the section of the graph from O to A should really look like?

Assuming that the dog was next to its owner at 0 s and had travelled 20 m in 5 s, we can be sure that the end points O and A are in the right place. However, the dog probably took a short time to achieve its maximum speed and had to slow down to reach a halt again.
The real graph might have looked like this:



Now let us consider the gradient of the straight line segment OA.
On a co-ordinate graph the gradient is calculated like this:

$$\text{gradient} = \frac{\text{change in } y}{\text{change in } x}, \quad \text{sometimes written as} \quad m = \frac{\Delta y}{\Delta x}$$

If we include the units in the calculation, we get:

$$\text{gradient OA} = \frac{20\text{ m}}{5\text{ s}} = 4\ \frac{\text{m}}{\text{s}} \quad \text{or} \quad 4\text{ m s}^{-1}$$

This is the answer we had previously for the speed of the dog.
Notice how including the units in the division gives us the units for the answer and therefore indicates how to interpret the outcome.

For distance–time graphs:

gradient = speed typical units: $\mathbf{m\,s^{-1}}$

We should really be more precise in our description of the result of 4 m s^{-1} which we calculated earlier as the 'speed of the dog'.
It would be more accurate to call this the ***average speed*** of the dog.
Comparing the straight line with the more realistic shape, we can see that the dog must have exceeded this speed in the middle of its outward run.

Average velocity and average speed:

$$\textbf{average velocity} = \frac{\textbf{change in displacement}}{\textbf{change in time}}$$

$$\textbf{average speed} = \frac{\textbf{change in distance}}{\textbf{change in time}}$$

Displacement–time graphs

When we are considering motion in a straight line, it is usually with respect to a fixed point. Sometimes this is simply the starting point for the motion. It is often referred to as an *origin* (which means beginning), in the same way as the origin on a graph.

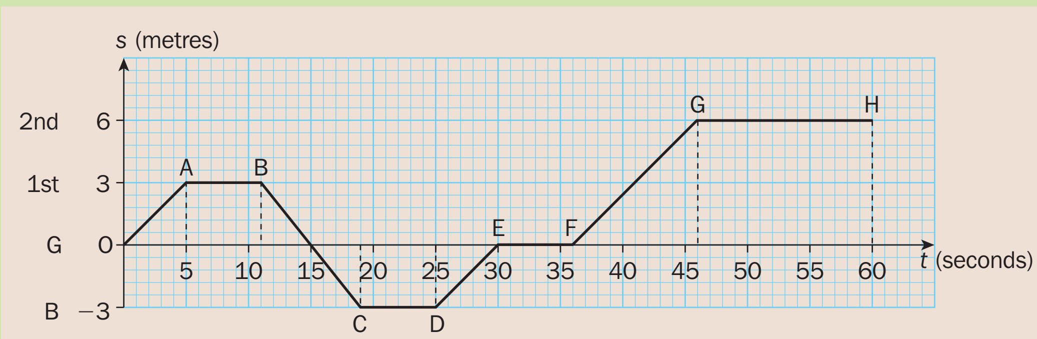
If an object can only move along a line, then there will be only two possible choices of direction.
We need to specify which way along the line will be the positive direction. The opposite way will then be the negative direction.

What effect will this have on a graph of displacement against time? Whereas distance is always positive, displacement can be negative so the vertical axis may need both positive and negative values.

Example 3

The graph below shows the displacement with respect to time of a lift moving in a vertical shaft between the basement level, ground floor and first and second floors.
Each level is 3 m apart.
Does the lift ascend at the same rate as it descends?

TO BE RESUPPLIED



We can compare the lines representing any ascent with section BC, which is the only descent.
Let us consider OA and BC. We already know that the velocity is found from the gradient of a line.

A line that rises as you follow it across to the right has a positive gradient. The line BC has a negative gradient because it comes down the page as you follow it in the direction of the horizontal t axis.
This corresponds to the direction in which the lift is travelling; from B to C it is descending.

$$\text{Gradient of OA} = \frac{3\text{ m}}{5\text{ s}} = 0.6\text{ m s}^{-1}, \qquad \text{Gradient of BC} = \frac{-6\text{ m}}{8\text{ s}} = -0.75\text{ m s}^{-1}$$

The signs of these velocities indicate their directions; 0.6 m s^{-1} upwards and 0.75 m s^{-1} downwards.
The speeds are simply the magnitudes: 0.6 m s^{-1} and 0.75 m s^{-1}.
The descent speed is therefore greater than the ascent speed.

For vertical motion, ground level is often taken as being the ‘zero’ level.
In the case of the lift in Example 3, it makes sense for the basement level to be ‘negative’ and the floors above to be ‘positive’.

TAKE IN PHOTO P3.8
Liff Picture
(outside building)
TO BE SUPPLIED

Once again, the graph of the motion has been simplified to produce straight lines. Can you imagine what a more precise graph might look like?
Think also about how the start and end of a journey in a lift actually feels.

Look again at the horizontal lines on the graph.
This time there is no doubt about what they represent; they show the times when the lift was stationary. In the straight-line vertical motion of the lift shaft there is no other possible way that the displacement from the ground floor level can be constant.

In this example the units were divided in the same way as the numbers, to confirm that the units for the answer would be correct for velocity.
In fact the calculation of the gradient on a displacement–time graph will always involve dividing a displacement by a time and therefore give an answer which will be a velocity.

For displacement–time graphs:

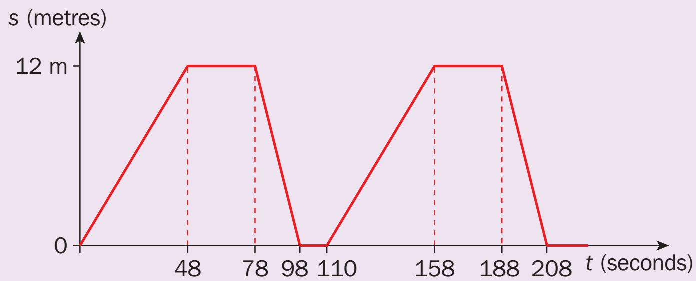
gradient = velocity typical units: $\mathbf{m\,s^{-1}}$

We do not need to continue to do the units check for this particular type of graph, but this method gives us a useful way of determining the meaning of the result of any calculation involving standard units.
This process is called **dimensional analysis**. We will use it again in the next section.

Practice

1 A cyclist leaves home at 10:00 am and cycles 1.6 km to the post office, arriving at 10:03 am. She then spends 5 minutes there before cycling on to the library. The library is a further 0.8 km and she arrives at 10:10 am. After spending 20 minutes in the library, she cycles directly home, reaching her house at 10:38 am. Show all this information on a distance–time graph and calculate the speed of the cyclist on each of her three journeys.

2 Here is a displacement–time graph of the vertical movement of a bucket on a rope being used to take bricks up to the top of a building. Briefly describe the motion and calculate the speed of the upwards and the downwards journeys.



Velocity–time graphs

We have seen how to work out speed and velocity from distance–time graphs and displacement–time graphs. It is also very common to show on a graph how the velocity changes against time.
What new information can we deduce from this sort of graph?

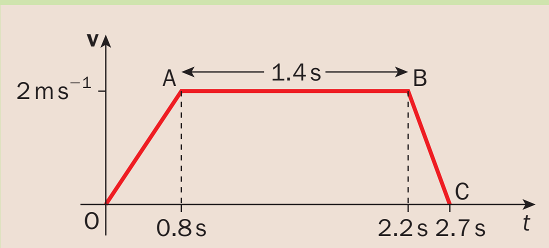
How does a velocity–time graph differ from a speed–time graph?
As the velocity tells us 'how fast and in which direction', it is this sense of the direction of the motion that makes the difference.
For any example where the object only moves in a forward direction, the graph against time of the velocity will be the same as it would be for speed.

Example 4

Christopher is driving his radio-controlled model car up the corridor.
It accelerates from rest to its top speed of $2\ \mathrm{m\,s^{-1}}$ in 0.8 s.
After travelling for 1.4 s at this speed, it brakes and comes to rest in 0.5 s.

TAKE IN PHOTO P3.9
radio-controlled car
TO BE SUPPLIED

Assuming smooth acceleration and deceleration, and that the direction is always up the corridor so that we know all the velocities are positive, the velocity–time graph would look like this:



a) What do the gradients of the three line segments tell us about the motion?
b) What is the area under the graph and what does it mean?

a) For OA: $\quad \text{gradient} = \dfrac{2\ \mathrm{m}}{\mathrm{s}} \div \dfrac{0.8\ \mathrm{s}}{1} = \dfrac{2\ \mathrm{m}}{\mathrm{s}} \times \dfrac{1}{0.8\ \mathrm{s}} = 2.5\ \mathrm{m\,s^{-2}}$

Since AB is horizontal, its gradient is zero.

For BC: $\text{gradient} = \dfrac{-2\ \mathrm{m}}{\mathrm{s}} \div \dfrac{0.5\ \mathrm{s}}{1} = \dfrac{-2\ \mathrm{m}}{\mathrm{s}} \times \dfrac{1}{0.5\ \mathrm{s}} = -4\ \mathrm{m\,s^{-2}}$

Once again, using the units in the calculation shows us that the gradient gives us the ***acceleration***.
For the centre section, the velocity is constant, so the acceleration is zero.
The negative acceleration, or ***retardation***, at the end shows that the car is slowing down.

b) The area under the graph is a trapezium. Area of a trapezium $= \left(\dfrac{a+b}{2}\right) \times h$

Area under the graph $= \left(\dfrac{2.7\ \mathrm{s} + 1.4\ \mathrm{s}}{2}\right) \times \dfrac{2\mathrm{m}}{\mathrm{s}} = \underline{4.1\ \mathrm{m}}$

This is the distance travelled by the toy car. As we know, this is all in the same (positive) direction, so we can say this is the total ***displacement*** of the car.

The speed of an object is the magnitude of its velocity and will therefore be positive.
As the velocity is always positive, the speed–time graph would look the same as the one above.

For velocity–time graphs:

gradient = acceleration typical units: $\mathbf{m\,s^{-2}}$

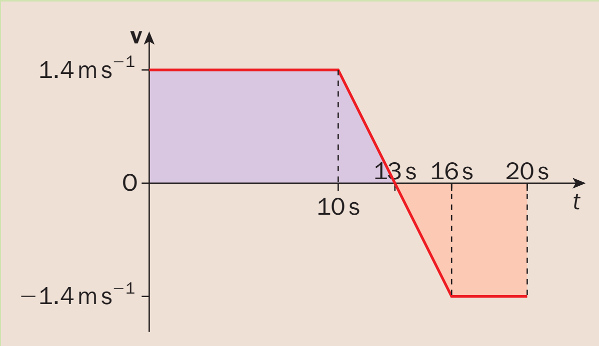
area = displacement typical units: **m**

Where the displacement is always positive, this will be the same as the distance travelled.

How will the graph look different for a situation where the movement is not always in the same direction? What if the object moves 'backwards'?

Example 5

During shunting manoeuvres in a siding, an engine is moving forwards at $1.4\,\mathrm{m\,s^{-1}}$ at $t = 0$ s. The engine driver applies the brakes after $t = 10$ s and, after coming to rest three seconds later, immediately accelerates in the opposite direction to reach a velocity of $-1.4\,\mathrm{m\,s^{-1}}$ by $t = 16$ s. This graph shows the velocity during the first 20 s of the motion of the engine. Find the area of each of the shaded sections and explain what it means.



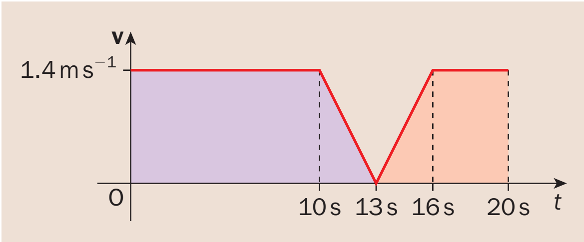
For the section for 0 to 13 seconds: area $= \left(\dfrac{10 + 13}{2}\right) \times 1.4 = +16.1\ \mathrm{m}$ (displacement, forwards)

For the section for 13 to 20 seconds: area $= \left(\dfrac{4 + 7}{2}\right) \times (-1.4) = -7.7\ \mathrm{m}$ (displacement, backwards)

(The area of a part of the graph below the t axis gives the distance travelled in the negative direction.)

The sum of these gives the total displacement: $16.1 + (-7.7) = 8.4$ m (forwards).
(This is not the same as the total distance travelled: $16.1 + 7.7 = 23.8$ m.)

The speed–time graph for the same example would look like this:



How does it differ from the velocity–time graph?
The main difference is that the direction of the velocity is ignored and only its magnitude is plotted, which will always be positive (or zero).
In effect, the parts of the graph underneath the horizontal axis have been reflected up, to appear above this axis.

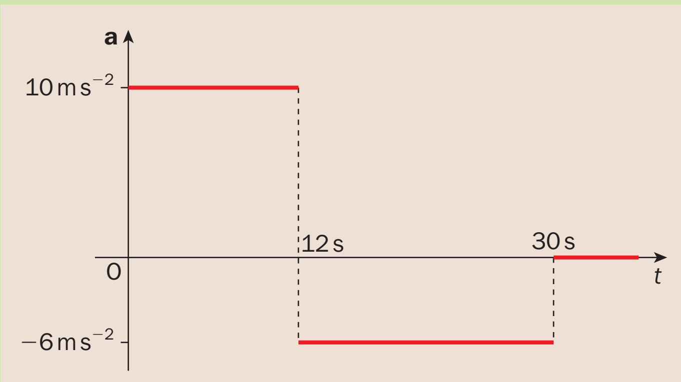
Acceleration–time graphs

Now that we have seen graphs of distance, displacement and velocity plotted against time, what can we learn from acceleration–time graphs? In fact these are far less commonly used, particularly when considering motion with constant acceleration.

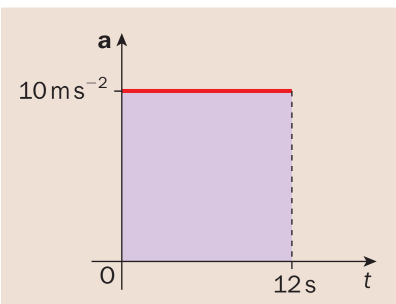
What do you expect an acceleration–time graph to look like?
If the acceleration is constant, it will consist of horizontal straight lines.
When the speed is constant and the acceleration is zero, there will be a flat line along the x axis.
When the object is accelerating, the line will be above the x axis.
When the object is decelerating, the line will be below the x axis.

Example 6

A sky-diver jumps out of a stationary hovering helicopter and free-falls for 12 s. His acceleration towards the ground is 10 m s^{-2}. He then releases his parachute and experiences a retardation of 6 m s^{-2}.
After a further 18 s he reaches a constant speed.
Taking downwards as the positive direction for motion and assuming he falls in a vertical straight line, sketch a graph of the acceleration of the sky-diver against time.



Taking the first section of the graph, from $t = 0$ to $t = 12$ s, what does the area under the graph tell us?



Carefully multiplying both the numbers and the units, as we did before, gives us:

$$\text{area} = 10\text{ m s}^{-2} \times 12\text{ s}$$
$$= 120\text{ m s}^{-1}$$

This is the speed reached by the sky-diver after the first 12 s.

For acceleration–time graphs:

area = change in speed typical units: $\mathbf{m\,s^{-1}}$

Use this principle to find the negative area between the t axis and the second section of the graph.
Use this to find the final speed of the sky-diver.
Did you get an answer of 12 m s^{-1}?

Summary

- **Speed** is the rate of change of **distance** with respect to time
- **Velocity** is the rate of change of **displacement** with respect to time
- **Acceleration** is the rate of change of velocity (or speed) with respect to time
- **Deceleration** (or **retardation**) is the magnitude of a negative acceleration
- **Average velocity** = $\dfrac{\textbf{change in displacement}}{\textbf{change in time}}$
- **Average speed** = $\dfrac{\textbf{change in distance}}{\textbf{change in time}}$
- For **distance–time graphs:** gradient = speed
- For **displacement–time graphs:** gradient = velocity
- For **velocity–time graphs:** gradient = acceleration, area = displacement
- For **acceleration–time graphs:** area = change in speed

End of Chapter Questions

1 Can you complete these sentences?

a) The horizontal axis usually represents …

b) Speed is the rate of change of … with respect to time.

c) … is the rate of change of displacement with respect to time.

d) The gradient on a velocity–time graph represents the …

e) The area under a velocity–time graph represents the …

f) The meaning of an area on a graph can be found by …

2 What does a horizontal line on a distance–time graph usually mean?

3 A fish darts 11 metres in 1.7 seconds. What is its speed?

4 A car travels a total distance of 100 miles in $2\frac{1}{4}$ hours. Convert this information to SI units and hence find the average speed of the car in $\mathrm{ms^{-1}}$.

5 What can we tell from a horizontal line on a velocity–time graph?

6 A police car accelerates from 0 to $20\,\mathrm{ms^{-1}}$ in 8 seconds. It then travels at this speed for 15 seconds. The brakes are applied and the vehicle comes to rest after a further 5 seconds. Sketch a speed–time graph and by finding the area underneath the graph, calculate the total distance the car has travelled.

7 A young girl climbs the steps up to the top of a 2.3 m high playground slide. This takes her 9 seconds. She waits at the top for 5 seconds and then slides down to the bottom, arriving 18.5 seconds after the start of the climb. Show this information on a displacement–time graph. Calculate the velocity of her descent.

8 exam

A boy kicks a football so as to hit a wall 10 m away at a right angle. The ball travels to the wall at a constant speed of $20\,\mathrm{m\,s^{-1}}$ and returns along the same line at a speed of $12\,\mathrm{m\,s^{-1}}$. The boy then traps the ball at a distance of 8 m from the wall.

a) Draw a displacement–time graph for the motion of the ball from when it was first kicked to when it was trapped, measuring the displacement from the point at which it was first kicked.

b) Find the average speed of the ball while it is in motion.

c) Find the average velocity of the ball while it is in motion.

9 exam A car starts from rest at a set of traffic lights and moves along a straight horizontal road with constant acceleration $4\ \mathrm{m\,s^{-2}}$ until it reaches a speed of $24\ \mathrm{m\,s^{-1}}$. It maintains this speed for 30 seconds before decelerating at $6\ \mathrm{m\,s^{-2}}$ until it comes to rest at the next set of traffic lights.

a) Draw a speed–time graph to describe the motion of the car.

b) Find the distance between the two sets of traffic lights.

c) Find the average speed of the car for the whole journey.

10 exam An athlete runs the 100 m along a straight horizontal track in 10.5 seconds. She accelerates uniformly from rest for 2 seconds to a top speed of $V\ \mathrm{m\,s^{-1}}$ which she then maintains until she crosses the finishing line. It then takes her 4 further seconds to decelerate uniformly to rest.

a) Illustrate the athlete's motion on a speed–time graph.

b) Find, to 1 decimal place, the value of V.

c) Find, to 1 decimal place, the total distance travelled by the athlete.

11 exam A body moving in a straight line accelerates from rest for 8 seconds at a constant rate of $2\ \mathrm{m\,s^{-2}}$. It then decelerates at a constant rate of $1\ \mathrm{m\,s^{-2}}$ for 14 seconds. It then travels at a constant speed for 5 seconds before decelerating uniformly to rest in a further 6 seconds.

a) Show the motion of the body on a speed–time graph.

b) Calculate the final deceleration of the body.

c) Calculate the total distance moved by the body.

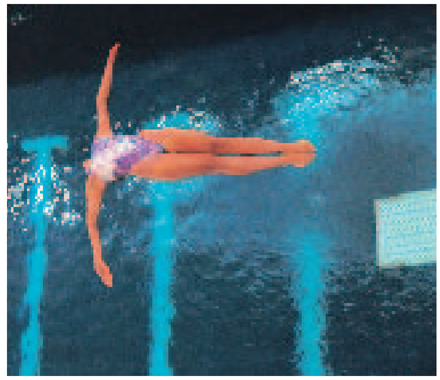
How to make the Examiner happy

- For any form of travel graph, label the scale on both axes and state the quantity as well as the units, e.g. "displacement (metres)"
- Don't be caught out by the units for distance; we use 'm' as the shorthand for both miles and metres. Expect it to be metres or kilometres unless the question makes it clear that it should be miles
- Each section of the graph will probably be a straight line. Make sure the endpoints of each section are in exactly the right place
- If you do a gradient calculation, show on the graph the section you are using and label the horizontal and vertical parts
- For any area calculation that involves more than one section, show the method clearly in stages, labelling the sections on the graph
- Make sure you put the right units on your answers for gradient or area

THIS CHAPTER IS ONE PAGE SHORT FOR DOUBLE PAGE SPREADS.
PLEASE ADVISE.

4 The Uniform Acceleration Formulae

For certain situations we can assume that the acceleration involved is constant. Can you think of some examples?
The best practical cases involve gravity, such as dropping an object or jumping off a diving board.
Sometimes we just assume that the acceleration is constant for a short time.



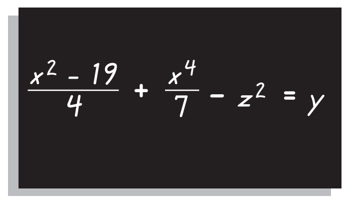
There are a number of equations which we can use if we know that the acceleration is constant. These are called the **uniform acceleration formulae**. ('Formulae' is the plural of 'formula'.)

In this chapter you will learn:

- how to substitute into formulae,
- how to rearrange formulae,
- how to convert units,
- how to select the most appropriate uniform acceleration formula to use,
- how to use vectors in uniform acceleration formulae.

Using formulae

What is the difference between 'equations' and 'formulae'?
The answer is 'very little!'.
An ***equation*** is a mathematical expression which includes an 'equals' sign.
A ***formula*** is usually an equation that gives a useful practical result.



This is a good time to practise substituting numbers into equations. There are a few useful 'tips' to remember and some 'pitfalls' to avoid.

As this chapter is focused on the uniform acceleration formulae, we will practise using these particular equations. You need to memorise them.

The uniform acceleration formulae (a.k.a. the SUVAT formulae):

$$v = u + at$$
$$s = ut + \tfrac{1}{2}at^2$$
$$s = vt - \tfrac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$
$$s = \tfrac{1}{2}(u + v)t$$

where:		
	s = displacement	(m, metres)
	u = initial speed	($\mathrm{m\,s^{-1}}$, metres per second)
	v = final speed	($\mathrm{m\,s^{-1}}$, metres per second)
	a = acceleration	($\mathrm{m\,s^{-2}}$, metres per second squared)
	t = time taken	(s, seconds)

If the information is given in any other units, it *must* be changed into these units first.

Distance, Speed and Time Triangle

Although the relationships summarised in this triangle are still true, make sure you learn the right letters to use in mechanics.
Distance ≡ s (displacement)
Speed ≡ *u* or v (initial/final speed or velocity)
Time ≡ *t* (time)

Example 1

Substitute $u = 5$, $a = 0.25$ and $t = 16$ into $v = u + at$ to find v.

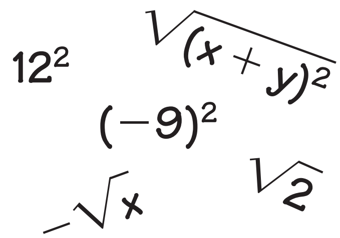
Write out the formula and then rewrite it underneath with the actual numbers replacing the letters. Then calculate the required answer.

$$v = u + at$$
$$v = 5 + (0.25).16$$
$$v = 5 + 4$$
$$v = \underline{9}$$

Did you notice these three things?

- In algebra, two letters together means they are multiplied. In this case: $at = a \times t$.
- When we substitute into equations involving multiplication, this is often shown using a dot rather than a '×' sign.
- However, where decimals are concerned, brackets are sometimes included to avoid any confusion. Otherwise 0.25×16 could look like 0.25.16 and be misread as $0 \times 25 \times 16$.

In general we would be given the units for the starting quantities and we should carefully put the correct units after the answer.
What would be the correct units for the answer $v = 9$?
It should be written as $v = 9\ \text{m s}^{-1}$.



What do you remember about 'squaring' and 'square rooting'?
The two important tips when using these formulae are that:

- anything 'squared' will end up *positive*,
- the square root of a number can be *positive* or *negative*.

Example 2

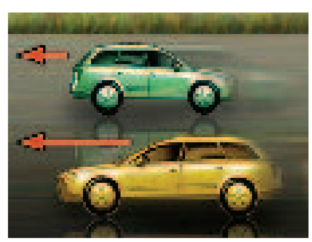
Use the formula $v^2 = u^2 + 2as$ to find a) v^2 and b) v, when $u = -4\ \text{m s}^{-1}$, $a = 2\ \text{m s}^{-2}$ and $s = 5$ s.

a) $v^2 = u^2 + 2as$
$$v^2 = (-4)^2 + 2.2.5$$
$$v^2 = 16 + 20$$
$$v^2 = \underline{36}$$

(Notice the brackets used around –4 to make it quite clear that $(-4)^2 = (-4) \times (-4) = 16$.)

b) $v^2 = 36$
$$v = \sqrt{36}$$
$$v = \underline{6\ \text{m s}^{-1} \text{ or } -6\ \text{m s}^{-1}}$$

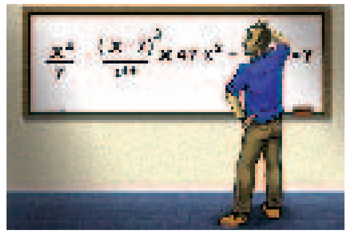
According to the formula *both* of these answers are possible.
Depending on the context of the question, *only one* of them might be appropriate.



What does a starting speed of $-4\ \text{m s}^{-1}$ mean?
The negative sign implies 'in the opposite direction' or **backwards**.

In example 2, the two answers both involve a speed of $6\ \text{m s}^{-1}$, but the two possible solutions to the problem are $6\ \text{m s}^{-1}$ forwards, or alternatively, $6\ \text{m s}^{-1}$ backwards.

Rearranging formulae



The two sides of an equation (i.e. either side of the 'equals' sign) must always balance. So, the golden rule for rearranging equations is:
do the same to both sides.

How do you know what operation to do to both sides?
Look at the change you are trying to make and do the 'inverse operation', i.e. **the mathematical opposite**, to both sides.

What does it mean if you are asked to 'make r the **subject** of the equation'?
This means that the equation needs rearranging so that it reads: $r = \ldots$

Example 3

Rearrange the equation $s = \frac{1}{2}(u + v)t$ to make u the subject.

$s = \frac{1}{2}(u + v)t$	double both sides, the inverse of 'halve'
$2s = (u + v)t$	divide both sides by t
$\dfrac{2s}{t} = (u + v)$	only now can we safely drop the brackets
$\dfrac{2s}{t} = u + v$	subtract v from both sides
$\dfrac{2s}{t} - v = u$	finished, but not quite in the form $u = \ldots$

$$\therefore \quad \underline{u = \frac{2s}{t} - v}$$

Notice how it is correct style to line up the 'equals' sign down the page, rather than to work from the left hand margin, as in ordinary writing.

Example 4

Rearrange the equation $v^2 = u^2 + 2as$ to make u the subject.

$v^2 = u^2 + 2as$	subtract the term $2as$ from both sides
$v^2 - 2as = u^2$	take the square root of both sides
$\sqrt{v^2 - 2as} = u$	rewrite to make it clear that u is the subject

$$\therefore \quad \underline{u = \sqrt{v^2 - 2as}}$$

Changing units

With the list of uniform acceleration formulae at the start of the chapter there is a note of the normal units that must be used for these formulae to work. If the measurements are given using any different units they must be converted first.

In what units might the displacement, s, be given?
If SI units (*Système International*, the usual metric units) are used, they could be metres, centimetres, millimetres or kilometres.
Imperial units used could be miles, yards, feet or inches.

In a given problem we may need to convert between different SI units.

Conversions
Only metric conversions need to be learnt for the examination.

Remember: **1 kilometre = 1000 metres**
1 metre = 100 centimetres
1 centimetre = 10 millimetres
1 metre = 1000 millimetres

Example 5
Convert a displacement of 16 cm into metres.

Using the relationship that 100 cm = 1 m, we see that we must divide by 100.

$$\begin{aligned} 16\text{ cm} &= 16 \div 100\text{ m} \\ &= \underline{0.16\text{ m}} \end{aligned}$$

(Remember to change the units at the same moment as you do the mathematical conversion.)

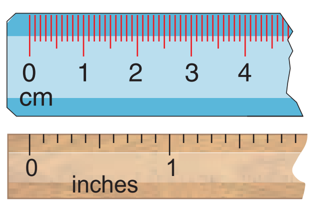
For compound units, the conversion process will involve more than one step.

Example 6
Convert 54 kilometres per hour into metres per second.

This will require two steps, converting the kilometres using 1 km = 1000 m and then converting the hours using 1 hour = 60 minutes = 60 × 60 seconds.

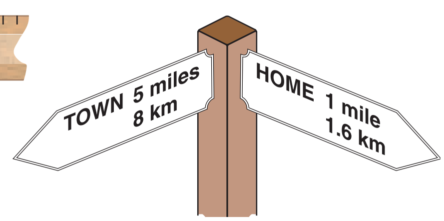
$$\begin{aligned} 54\text{ km.p.h.} &= 54 \times 1000\text{ metres per hour} \\ &= 54\,000\text{ metres per hour} \\ &= 54\,000 \div (60 \times 60)\text{ metres per second} \\ &= 54\,000 \div 3600\text{ m s}^{-1} \\ &= \underline{15\text{ m s}^{-1}} \end{aligned}$$

If we need to convert between Imperial and metric units, special conversion factors are required.



How many centimetres are in one inch?
2.54 cm = 1 inch.

What is the connection between miles and kilometres?
5 miles = 8 kilometres or 1 mile = 1.6 km.



Do you know any other conversions? See if you can find some more.

Choosing the right formula

Have you wondered why there are five different SUVAT formulae?
Count how many variables appear in each one.
Now work out which variable is missing from each one.

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Each one of the five formulae connects *four* of the variables.
The five formulae listed at the start of the chapter each exclude a different one of the five SUVAT variables.

If we look at the information given in the question and which quantity is required, we can choose the right formula to use by deciding which quantity is *not* involved.

The uniform acceleration formulae (a.k.a. the SUVAT formulae):
This table shows with a cross which variable is missing from each equation:

	s	u	v	a	t
$v = u + at$	✗	✓	✓	✓	✓
$s = ut + \frac{1}{2}at^2$	✓	✓	✗	✓	✓
$s = vt - \frac{1}{2}at^2$	✓	✗	✓	✓	✓
$v^2 = u^2 + 2as$	✓	✓	✓	✓	✗
$s = \frac{1}{2}(u + v)t$	✓	✓	✓	✗	✓

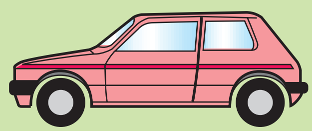
Example 7
Which formula should be used to find u, if we know s, v and t?

Think of the 'word' SUVAT. The letter that isn't mentioned in the question is a.

Looking at the table above, that means we need the equation: $\underline{s = \frac{1}{2}(u + v)t}$

It always helps to list all the known information at the start of a question and to make a note of the required quantity.
If the question is worded as a problem in context, pick out and list the measurements that are given.

Example 8
A car decelerates at a rate of $2\ \mathrm{m\,s^{-2}}$.
What distance will it have covered after 8 s,
if its speed is initially $30\ \mathrm{m\,s^{-1}}$?



First, list the quantities in the question:

$$a = -2\ \mathrm{m\,s^{-2}}, \quad t = 8\ \mathrm{s}, \quad u = 30\ \mathrm{m\,s^{-1}}, \quad s = ?$$

As v does not appear in the question, we need to use:

$$s = ut + \tfrac{1}{2}at^2$$
$$s = 30 \times 8 + \tfrac{1}{2} \times (-2) \times 8^2$$
$$s = 240 - 64$$
$$s = \underline{176\ \mathrm{m}}$$

Vector equations

We can use the uniform acceleration formulae in two or three dimensions as well, by writing the different quantities as vectors.

There are three things to remember:

- underline the letters in the formulae representing vectors,
- t (time) is *not* a vector quantity,
- don't use $v^2 = u^2 + 2as$.

$\cancel{v^2 = u^2 + 2as}$

What is the problem with using $v^2 = u^2 + 2as$?
It involves squaring and multiplying vectors. The other formulae only require adding, subtracting or scalar multiples of vectors.

Examination questions will normally be set using unit vectors. If you choose to use column vector notation in your solution, make sure to give your final answer in the same format as the question.

Example 9

A particle travelling with initial velocity $(3\mathbf{i} + 7\mathbf{j})\ \mathrm{m\,s^{-1}}$ experiences an acceleration of $(2\mathbf{i} - \mathbf{j})\ \mathrm{m\,s^{-2}}$. What will be its new velocity after 15 seconds?

List the quantities in the question, remembering to use proper vector notation.
(For handwritten solutions this means underlining all vector quantities.)

$$\mathbf{u} = (3\mathbf{i} + 7\mathbf{j})\ \mathrm{m\,s^{-1}}, \quad \mathbf{a} = (2\mathbf{i} - \mathbf{j})\ \mathrm{m\,s^{-2}}, \quad t = 15\ \mathrm{s}, \quad \mathbf{v} = ?$$

The equation linking these is:

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$
$$\mathbf{v} = (3\mathbf{i} + 7\mathbf{j}) + (2\mathbf{i} - \mathbf{j}) \times 15$$
$$\mathbf{v} = 3\mathbf{i} + 7\mathbf{j} + 30\mathbf{i} - 15\mathbf{j}$$
$$\mathbf{v} = 33\mathbf{i} - 8\mathbf{j}$$

The new velocity of the particle is: $\underline{(33\mathbf{i} - 8\mathbf{j})\ \mathrm{m\,s^{-1}}}$

Example 10

Find $\mathbf{s}$ if the initial velocity is $\begin{pmatrix} 7 \\ 12 \\ -5 \end{pmatrix}\ \mathrm{m\,s^{-1}}$ and, 12 seconds later, the new velocity is $\begin{pmatrix} 4 \\ -4 \\ 0 \end{pmatrix}\ \mathrm{m\,s^{-1}}$.

$$\mathbf{u} = \begin{pmatrix} 7 \\ 12 \\ -5 \end{pmatrix}\ \mathrm{m\,s^{-1}}, \quad \mathbf{v} = \begin{pmatrix} 4 \\ -4 \\ 0 \end{pmatrix}\ \mathrm{m\,s^{-1}}, \quad t = 12\ \mathrm{s}, \quad \mathbf{s} = ?$$

As the acceleration is not involved, use:

$$\mathbf{s} = \tfrac{1}{2}(\mathbf{u} + \mathbf{v})t$$
$$\mathbf{s} = \tfrac{1}{2}\left(\begin{pmatrix} 7 \\ 12 \\ -5 \end{pmatrix} + \begin{pmatrix} 4 \\ -4 \\ 0 \end{pmatrix}\right) \times 12$$
$$\mathbf{s} = 6\begin{pmatrix} 11 \\ 8 \\ -5 \end{pmatrix}$$
$$\underline{\mathbf{s} = \begin{pmatrix} 66 \\ 48 \\ -30 \end{pmatrix}\ \mathrm{m}}$$

Two-step vector problems

How can we avoid having to use $v^2 = u^2 + 2as$ to solve problems in two or three dimensions?
We will have to use one of the other equations to find out first the one quantity we *don't* need, the time, t.
Then we can select another equation to obtain the required result.

Examination Requirements
At present, this two-step method will not be included in the examination. Only problems that can be solved using one equation directly will be set.

Example 11

Find the initial velocity of a particle if, after accelerating at $(2\mathbf{i} + \mathbf{j})\ \text{m s}^{-2}$ over a total displacement of $(120\mathbf{i} + 320\mathbf{j})$ m, its final velocity is $(26\mathbf{i} + 26\mathbf{j})\ \text{m s}^{-1}$.

Summarising the question: $\mathbf{a} = (2\mathbf{i} + \mathbf{j})\ \text{m s}^{-2}$, $\mathbf{s} = (120\mathbf{i} + 320\mathbf{j})$ m, $\mathbf{v} = (26\mathbf{i} + 26\mathbf{j})\ \text{m s}^{-1}$, $\mathbf{u} = ?$

As we can't find $\mathbf{u}$ directly, we must use $\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$ to find t instead:

$$120\mathbf{i} + 320\mathbf{j} = (26\mathbf{i} + 26\mathbf{j})t - \tfrac{1}{2}(2\mathbf{i} + \mathbf{j})t^2$$

$$120\mathbf{i} + 320\mathbf{j} = 26t\mathbf{i} + 26t\mathbf{j} - t^2\mathbf{i} - \tfrac{1}{2}t^2\mathbf{j}$$

Consider first the $\mathbf{i}$ components: $120 = 26t - t^2$

Rearranging this gives: $t^2 - 26t + 120 = 0$

Factorising: $(t - 20)(t - 6) = 0$

$t = 20$ or $t = 6$ … but which value do we use?

Now consider the $\mathbf{j}$ components: $320 = 26t - \frac{1}{2}t^2$

Doubling this gives: $640 = 52t - t^2$

Rearranging: $t^2 - 52t + 640 = 0$

Factorising: $(t - 32)(t - 20) = 0$

$t = 32$ or $t = 20$

$\therefore$ The only value that will work for both the $\mathbf{i}$ and $\mathbf{j}$ components is $t = 20$ s.

The easiest equation to find $\mathbf{u}$ is: $\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$$26\mathbf{i} + 26\mathbf{j} = \mathbf{u} + (2\mathbf{i} + \mathbf{j}) \times 20$$

$$26\mathbf{i} + 26\mathbf{j} = \mathbf{u} + 40\mathbf{i} + 20\mathbf{j}$$

$$-14\mathbf{i} + 6\mathbf{j} = \mathbf{u}$$

$$\therefore \quad \mathbf{u} = -14\mathbf{i} + 6\mathbf{j}$$

The initial velocity of the particle is: $\underline{(-14\mathbf{i} + 6\mathbf{j})\ \text{m s}^{-1}}$

When finding the value of t from the information given, we chose the only value that worked for both the $\mathbf{i}$ component and the $\mathbf{j}$ component.

Sometimes solving for t can give us a positive and a negative solution. In such cases, we can usually *ignore* any negative values for t as we normally assume the motion did not start until $t = 0$ s. So only positive values of t apply to a given problem.

Summary

- These equations must be memorised:

$$v = u + at$$
$$s = ut + \tfrac{1}{2}at^2$$
$$s = vt - \tfrac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$
$$s = \tfrac{1}{2}(u + v)t$$

- **List** the quantities in the question.
- Check all the information is given in **SI units**; convert the information if necessary.
- Find the letter missing from **SUVAT** to decide which equation to use.
- For two-dimensional or three-dimensional problems, use the formulae in **vector** form.
- Instead of using $v^2 = u^2 + 2as$ in vector problems, first find t using another equation.

End of Chapter Questions

1 Can you complete these sentences?

a) To use these equations, the acceleration must be …

b) Make sure all the information is given in … units.

c) 'Make t the subject of the equation' means it must be written as …

d) At the start, it is a good idea to … all the information given.

e) When we use vectors, the one equation we can't use is …

f) It may be necessary to find … first, when using vectors.

2 Using $s = ut + \frac{1}{2}at^2$, find s when $u = 12\ \text{m s}^{-1}$, $t = 30$ s and $a = 0.4\ \text{m s}^{-2}$.

3 Find the two possible values of u, when $v = 60\ \text{m s}^{-1}$, $s = 256$ m and $a = -2\ \text{m s}^{-2}$.

4 Make t the subject of the formula $v = u + at$.

5 Convert a speed of 24 kilometres per hour into metres per second.

6 If you know the values of t, v and a, which formula would you use to work out s?

7 A particle is initially at rest and after 40 seconds is travelling at a speed of $60\ \text{m s}^{-1}$. Calculate the distance it has travelled.

8 A vehicle accelerates from 22 km.p.h. to 50 km.p.h. in a time of 10 seconds. Calculate the acceleration of the vehicle.

9 Find the total displacement when a particle accelerates at $\begin{pmatrix} 3 \\ 4 \end{pmatrix}\ \text{m s}^{-2}$ for 5 s, given that its initial velocity is $\begin{pmatrix} -12 \\ 17 \end{pmatrix}\ \text{m s}^{-1}$.

10 A particle has an initial velocity of $(10\mathbf{i} + 15\mathbf{j})\ \text{m s}^{-1}$ and a final velocity of $(-10\mathbf{i} + 45\mathbf{j})\ \text{m s}^{-1}$. If the total displacement of the particle is $300\mathbf{j}$ m, find its acceleration.

11 A particle moving in a straight line accelerates uniformly from $3\ \text{m s}^{-1}$ to $7\ \text{m s}^{-1}$ in 12 seconds. Find the distance travelled by the particle.

12 A car needs 1200 m of straight road to decelerate uniformly from its maximum speed of $60\ \text{m s}^{-1}$ to rest. Find

a) the deceleration of the car,

b) how long it will take to stop.

13 A car is moving along a straight road at $54\ \text{km hr}^{-1}$. It then accelerates uniformly for 10 seconds up to a speed of $72\ \text{km hr}^{-1}$.

a) Express $54\ \text{km hr}^{-1}$ in m s^{-1}.

Find

b) the acceleration of the car in $\mathrm{m\,s^{-2}}$,

c) the distance travelled by the car as it accelerates.

14 exam A particle P, moving in a straight line, passes the point O with velocity $7\ \mathrm{m\,s^{-1}}$. The particle then moves with a constant deceleration of $4\ \mathrm{m\,s^{-2}}$ until it returns to O. Given that at time T seconds P is 5 m from O,

a) show that T satisfies the equation $2T^2 - 7T + 5 = 0$.

b) Hence, or otherwise, find the length of time for which P is less than 5 m from O.

15 exam A car travels along a straight road with constant acceleration. It passes a point P and 3 seconds later passes a point Q where $PQ = 24$ m. After a further 2 seconds it passes a point R where $QR = 26$ m.

a) Find the acceleration of the car.

b) Find the speed when it passes P.

16 exam A train, starting from rest, accelerates for 90 s on a straight track, covering a distance of 729 m. It then maintains a constant speed for 600 s.

a) Find the acceleration of the train.

b) Find the total distance travelled by the train.

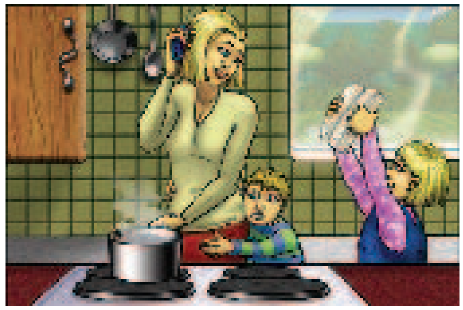
How to make the Examiner happy

- Make sure all the quantities are in SI units. If they are not, do the conversions before starting to use the formulae.
- Justify using these equations with a phrase such as “Since the acceleration is constant, we can use …”.
- List the known quantities and show clearly which one is required, e.g. “v = ?”.
- Make sure you state the units with your answer.
- For vertical motion, state clearly which is the positive direction! If you choose upwards to be positive, then the acceleration due to gravity will be $-9.8\ \mathrm{m\,s^{-2}}$.
- For questions where gravity is involved, take the value on the front of the exam paper, unless the question says otherwise. Don’t just use $9.8\ \mathrm{m\,s^{-2}}$ or $10\ \mathrm{m\,s^{-2}}$ without checking.

THIS CHAPTER IS ONE PAGE SHORT FOR DOUBLE PAGE SPREADS.
PLEASE ADVISE.

5 Dynamics

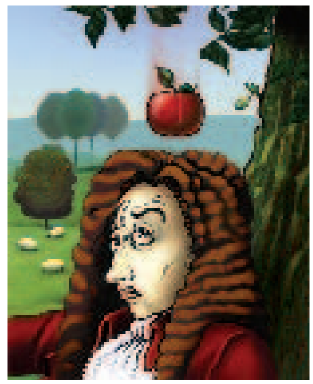
We say that a person is 'dynamic' if they have energy and drive. They can get things done or change things for the better.
In mechanics, the study of **dynamics** is all about the involvement of forces in a situation and how they produce or change the motion of an object.



Sometimes the way forces combine will prevent there being any movement.
Do you remember what this branch of mechanics is called?
It is called **statics**.

We have now built up a number of techniques for modelling reality. In this chapter these will be integrated with the application of forces, using the mathematics of vectors to represent the forces.

The main contribution to our model will come from Sir Isaac Newton, whose laws have established the basis for modern mechanics. It is even known as 'Newtonian Mechanics' in his honour.
Do you know the story about Newton and the apple?



Isaac Newton is said to have 'discovered' gravity when an apple fell on his head!

In this chapter you will learn:
- some of the different kinds of forces,
- more about resolving forces into perpendicular components,
- what forces resist motion,
- Newton's Three Laws of Motion,
- how to apply Newton's Laws to a variety of problems,
- how to use resolved components of forces for motion on a slope,
- how to model the movement of connected particles.

Forces

There are lots of different kinds of forces, so it is easiest to give a general definition in terms of what they all do.

A **force** will have one of the following effects:
- a force can speed up or slow down a moving object
- a force can prevent an object from moving
- a force can change the direction of a moving object.

TO BE
RESUPPLIED

There will always be a direction associated with the action of any force.
What branch of algebra deals with quantities that have both direction and magnitude?
You will remember that this is the defining quality of vectors.
We therefore use vectors to model forces, adding them or splitting them into components according to the rules of vectors.

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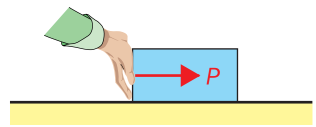
There are two main categories of forces:

contact forces and **non-contact forces**

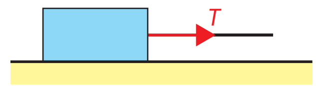
Can you think of some examples of each of these kinds of forces?

Contact forces

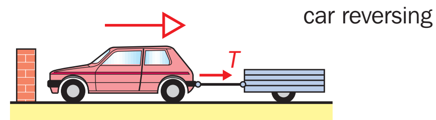
If an object is pushed or pulled by hand, that is clearly an example of a contact force.



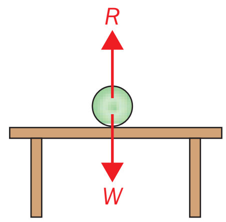
Sometimes the pulling force is in contact with the object through a cable or string. In this case the force is called **tension**.



If the force is pushing, a cable or wire would collapse. However, a solid connection such as a tow-bar could deliver a pushing force. The force would be called **thrust** in this case.



Any object that is in contact with another object or a surface is experiencing the **normal reaction force**.
This is a contact force and will be at right angles to the surface in question.



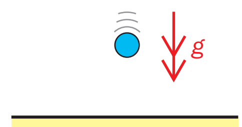
Non-contact forces

There are also forces of attraction which do not require physical contact. The most common of these which we all experience every day is due to **gravity**. We sometimes call it ‘the force of gravity’.
Which direction does it act in?
Gravity acts towards the centre of the Earth. But for all every-day problems, we simply consider it to act straight downwards.

‘Gravity’ (g) is the name for the acceleration of 9.8 m s^{-2} towards the centre of the Earth. (Sometimes, to simplify the calculation, we take $g = 10$ m s^{-2}.)

> *The value of g*
> Although some questions and examples in this book take $g = 10$ m s^{-2} for ease of calculation, it should be noted that in examination questions $g = 9.8$ m s^{-2} will always be used.

The force associated with gravity is **weight**. Although we all experience the same acceleration, everyone’s weight is different, depending on his or her mass.
The simple calculation is: weight = mass × g



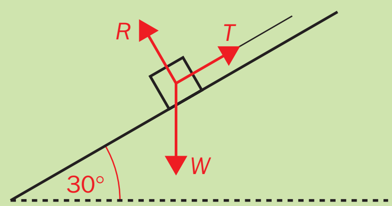
Gravity:

Acceleration due to gravity:	$g =$ **9.8 m s^{-2}**	(near the Earth’s surface)
Weight force:	mg **N**	(where m is the mass in kg)

The SI units of force are ‘**newtons**’ (N). The newton is a compound unit equivalent to kg m s^{-2}.

Example 1

A crate of mass 15 kg is resting on a slope which is inclined at an angle of 30° to the horizontal.
It is prevented from sliding because it is tied by a rope to a pole further up the slope.
Show the forces involved on a sketch, showing the direction and nature of each force.
Calculate the weight of the crate.



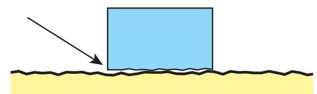
R = normal reaction force
T = tension in the rope
W = weight

If we take $g = 9.8$ m s^{-2}, the weight can be calculated to be $15 \times 9.8 =$ 147 N.

Resistance

In the Second World War, the French Resistance actively opposed the Nazi occupation of their country.
In mechanics, **resistance forces act to oppose motion**.
These forces will be in the opposite direction to that in which an object is moving, or is likely to move.

Friction



An object that is sliding along a surface is slowed down by the action of friction. Surfaces that are rougher produce more friction.
(There will be more about friction in chapter 7.)

Air resistance

Any object moving through the air is likely to be affected by **air resistance**. In practice, we will tend to ignore this effect for slow-moving objects. However, the faster a vehicle travels, the greater the effect of air resistance. How is this evident in the design of sports cars?
They tend to be sleeker and more streamlined, closer to the ground and have 'spoiler fins' to use this force to help them 'hug the ground'.

You may remember that we can ignore air resistance for a certain type of model. Which one was that?
Air resistance depends on both speed and surface area, so we ignore it for a *particle model*, which is assumed to have no surface area.

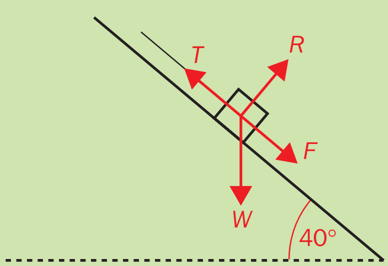
A resistive medium

Imagine a peanut is dropped into a pint of beer. It will fall to the bottom more slowly than if the glass was empty. There is an upward resistance force acting to reduce the downward speed.

What is the connection between this situation and air resistance?
An object moving through air is experiencing exactly the same sort of force, except that the molecules in the air are less densely packed.

Example 2

A schoolboy is hauling his lunchbox up the playground slide using a piece of string.
Assuming the slide is straight and inclined at 40° to the ground, show on a sketch all the following forces, considering their directions carefully: tension, weight, reaction, friction.



Friction is acting down the slide because the lunchbox is moving upwards.

Newton’s First Law

Newton was a man of rare insight. He lived in the 17th century, when anyone could tell you that to keep an object moving, you had to keep pushing! Even if it was sliding downhill it would still stop at the bottom! Newton, however, was able to recognise all the resistive forces at work and state that without these an object should keep moving indefinitely.

Sir Isaac Newton (1642–1727), English physicist and mathematician

> **Newton’s First Law:**
>
> **A body will continue to remain at rest or move at constant speed in a straight line unless an external force makes it act otherwise.**

These days we are familiar with seeing film from inside a space capsule or perhaps from the international space station. What happens when an object is passed from one astronaut to another?
It moves in a straight line at a constant speed.
Newton managed to come up with this law without seeing footage like this!

TAKE IN PHOTO P5.4
object floating between
astronauts
TO BE SUPPLIED

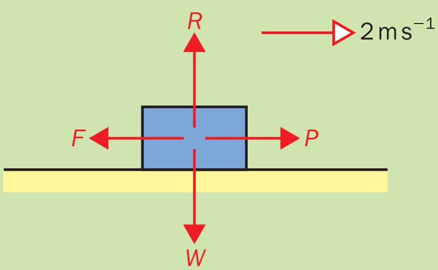
If the forces acting on a body ‘balance out’ then it will not experience any acceleration. It will remain stationary or, if it was already moving, will carry on at the same constant speed. Either way the acceleration is zero.

> **For constant speed (or to remain stationary):**
>
> The resultant force acting on a body will be zero.

In order to apply this as a method we simply need to show that the total of the forces acting equals zero overall.
One way is to split all the forces into components in two perpendicular directions. These will often be horizontal and vertical. We can then show by adding or equating that the total resultant force in each direction is zero. If the forces balance in both directions, the overall resultant force acting must also be zero.

Example 3

For the body shown below, state which forces must be equal in size for the body to continue to travel at a constant speed of $2\ \mathrm{m\,s^{-1}}$ in the direction indicated.



Since there is no acceleration, the total force in any direction is zero.

Horizontally (H): $\underline{P = F}$

Vertically (V): $\underline{R = W}$

(The letters H and V are often used instead of the words ‘horizontally’ and ‘vertically’ in written solutions to mechanics problems.)

Resolving forces on an inclined plane

Where the majority of the forces acting are either parallel or perpendicular to an inclined plane (slope), it is easiest to resolve any other forces into components in these directions.
If the object is *moving* along a slope we must definitely do this.

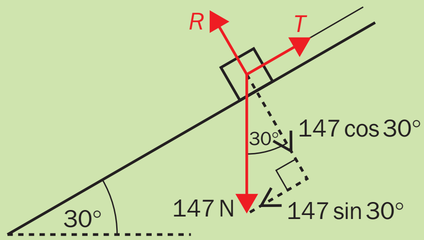
Look back at example 1. We calculated the weight, but not the other two forces acting. We will find them using this method.

Example 4

Here is the diagram of the three forces acting in example 1. The crate is stationary.
By resolving the weight into two components parallel and perpendicular to the slope, work out the magnitude of the reaction and the tension forces. Take $g = 9.8\ \text{m s}^{-2}$.

We previously calculated the weight of the crate to be 147 N.

We can use sine and cosine to find the components of the weight.
As the crate is stationary, the resultant force in any direction must be zero.



Perpendicular to the plane ($\perp$): $\quad R = 147 \cos 30° = 127.3\ \text{N}$

Parallel to the plane ($/\!/$): $\quad T = 147 \sin 30° = 73.5\ \text{N}$

It makes best sense to choose to resolve the weight into components parallel and perpendicular to the slope, because the other two forces are in these directions already.
If we chose horizontal and vertical components, we would have to resolve both of the other forces.

Forces in unit vector form

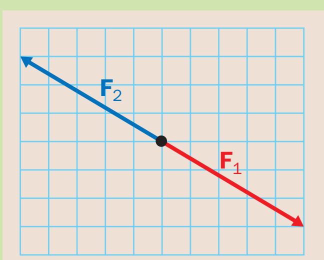
If the force is expressed in unit vector form, it is already in perpendicular components and we must show the total vector is $\mathbf{0}$ or $0\mathbf{i} + 0\mathbf{j}$.

Example 5

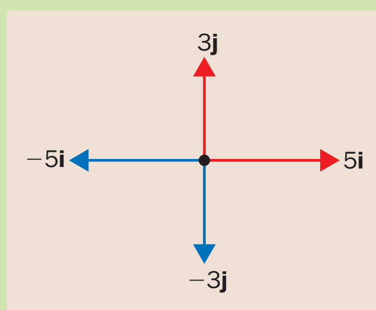
Two forces act on a particle which is moving with constant speed.
If one force, $\mathbf{F}_1$, is $(5\mathbf{i} - 3\mathbf{j})$ N, what is the other force, $\mathbf{F}_2$? Show your solution on a sketch.

The two forces must add up to the zero vector: $\quad \mathbf{F}_1 + \mathbf{F}_2 = \mathbf{0}$
Making the total force in each direction zero: $\quad (5\mathbf{i} - 3\mathbf{j}) + \mathbf{F}_2 = 0\mathbf{i} + 0\mathbf{j}$.
We can see that $\mathbf{F}_2 = (-5\mathbf{i} + 3\mathbf{j})$ N, since: $\quad (5\mathbf{i} - 3\mathbf{j}) + (-5\mathbf{i} + 3\mathbf{j}) = 0\mathbf{i} + 0\mathbf{j}$.

Sketching this:



or in components:



Either way, we can see that the resultant force acting is zero.

Practice

Throughout this exercise, take $g = 9.8\ \text{ms}^{-2}$.

1 What is the weight of a car of mass 950 kg?

2 What is the weight of a small ball of mass 88 g?

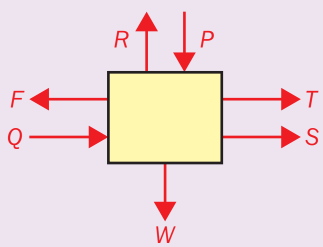
3 For which of these three objects should we **not** ignore air resistance: a bullet on its path to the target, a tennis ball in flight, a shuttle cock in flight.

4 Draw a force diagram for each of these situations, labelling the forces clearly;

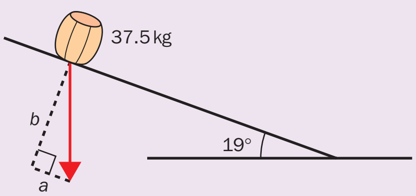
a) A woman pushing a loaded shopping trolley at a constant speed

b) A man pushing a cart up a slight incline.

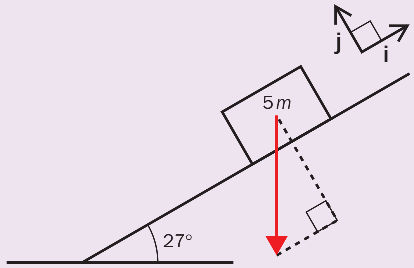
5 The object in the diagram remains at rest under the action of the forces shown. Write equations to show how the forces must balance.



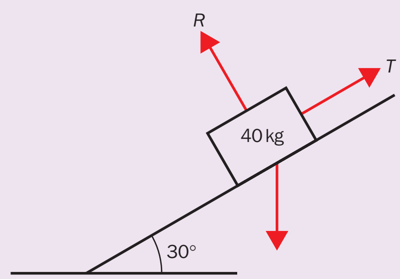
6 A barrel of mass 37.5 kg is resting on a plane inclined at 19° to the horizontal. Express the weight in components as shown.



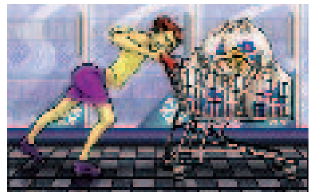
7 Taking the unit vectors to be acting in the directions shown in the diagram, express the weight of the box in vector form in terms of m.



8 A crate of mass 40 kg lies at rest on a plane inclined at 30° to the horizontal direction. Calculate the magnitude of the forces R and T as shown on the diagram.



Newton's Second Law



We all know from experience that a greater force is needed to start a full supermarket trolley moving than was required when it was empty.
What is the connection between force and acceleration?

Newton's observations on this subject led to the following rule, which can also be expressed in a very clear equation form.

> **Newton's Second Law:**
>
> **A resultant force acting on a body produces an acceleration which is proportional to the resultant force.**

Do you remember the symbol that is used for proportionality?
We can use it to express the relationship between **F**, the resultant force, and **a**, the acceleration.

We would write: $\mathbf{F} \propto \mathbf{a}$

This means: $\mathbf{F} = k\mathbf{a}$, where k is the constant of proportionality

If we use SI units: $\mathbf{F} = m\mathbf{a}$

This could be rearranged as follows: $\dfrac{\mathbf{F}}{m} = \mathbf{a}$

This makes sense of our previous observations, as a constant force applied to two different masses will produce different accelerations.
The same force divided by a bigger mass will result in a smaller acceleration.

> **Newton's Second Law (in equation form):**
>
> In SI units: $\mathbf{F} = m\mathbf{a}$

In this vector equation, **F** stands for the resultant force acting in the same direction as the acceleration, **a**.

How can we tell from the equation that **F** and **a** are in the same direction?
Do you remember what we discovered in chapter 2 about parallel vectors?
We saw that a scalar multiple of a vector will be parallel to the original vector and the scalar will show how much bigger the length will be.
In Newton's Second Law, m is a positive multiple, so **F** and **a** must be parallel and in the same direction.

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The general method will be:
- resolve all the forces into components parallel and perpendicular to the direction in which motion is likely to take place,
- in the direction of motion, apply $\mathbf{F} = m\mathbf{a}$,
- equate the components in the other direction.

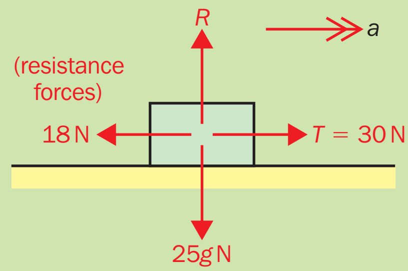
Example 6

An object with a mass of 25 kg is pulled along a flat horizontal surface by a taut horizontal cable with a tension of 30 N, against resistance forces totalling 18 N. (Take $g = 10\ \mathrm{m\,s^{-2}}$.)

a) Find the overall force acting i) vertically, and ii) horizontally.

b) Hence find the acceleration of the object, and state its direction.

First, draw a sketch of the situation showing all known quantities.



a) Consider the forces acting in each direction.

V: since there is no movement in a vertical direction, the vertical forces must be equal.

$R = 25g$ N

$R = 25 \times 10$

$R = \underline{250\ \mathrm{N}}$

H: the tension in the cable is greater than the resistance forces. The resultant force will be:

$30 - 18 = \underline{12\ \mathrm{N}}$ (to the right)

b) H: Using $F = ma$ (we can use just the magnitudes as the direction has been identified)

$12 = 25 \times a$

$12 \div 25 = a$

$a = 0.48\ \mathrm{m\,s^{-2}}$

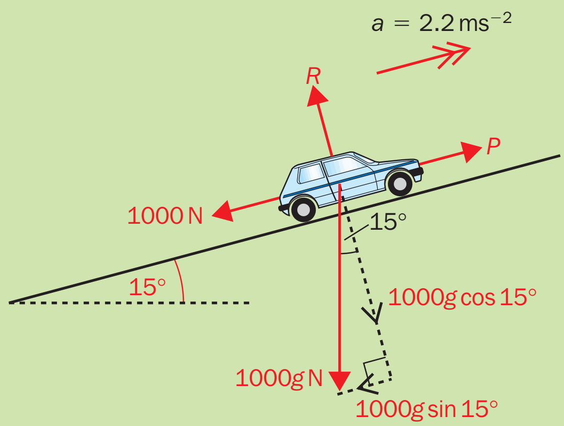
∴ The object will accelerate at a rate of $\underline{0.48\ \mathrm{m\,s^{-2}}}$, in the same direction as the tension in the cable.

The principle also applies to motion taking place on an inclined plane.
The forces perpendicular to the plane must balance.
Parallel to the plane we can use $\mathbf{F} = m\boldsymbol{a}$.

Example 7

A car of mass 1 tonne is travelling up a slope angled at 15° to the horizontal.
It is accelerating at $2.2\ \mathrm{m\,s^{-2}}$. Taking the value of g to be $9.8\ \mathrm{m\,s^{-2}}$, find the forward force produced by the engine if the resistance forces are a constant 1000 N.

Once again, sketch the situation.
The weight force needs to be resolved into components parallel and perpendicular to the plane of the slope.
(NB: 1 tonne = 1000 kg)



$\perp$ to the plane: $R = 1000g \cos 15°$

$R = 1000 \times 9.8 \times \cos 15°$

$R = 9470$ N (3 s.f.)

// to the plane: Using $F = ma$, where F is the magnitude of the resultant force

$P - 1000g \sin 15° - 1000 = 1000a$

$P = 1000 \times 2.2 + 1000g \sin 15° + 1000$

$P = 2200 + 2540 + 1000$

$P = 5740$ N (3 s.f.)

∴ The engine will produce a force of $\underline{5740\ \mathrm{N}}$

Newton's Third Law

When a force is applied, it provokes a response in the form of an equal force in the opposite direction.
Place this book in the middle of a table. The weight of the book is acting on the table. What is the opposite force?
It is the Normal Reaction force acting upwards on the book.
This force is stopping the book from moving downwards under the influence of gravity. It is one example of Newton's Third Law.

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Newton's Third Law

For every action there is an equal and opposite reaction.

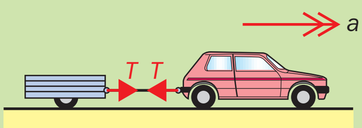
A simple way of applying this to modelling problems is to remember that: 'forces usually come in equal and opposite pairs'.
When we draw a force diagram, we need to add forces in equal and opposite pairs, as appropriate.

In practice we tend to focus on the forces that are acting on one object in particular, but it is still good practice to show how the pairs of forces act.

Example 8

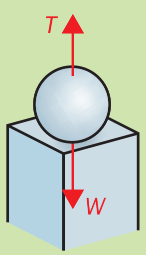
For each of the situations outlined below, describe the equal and opposite forces acting.

a) A tow bar connects an accelerating car to a trailer.



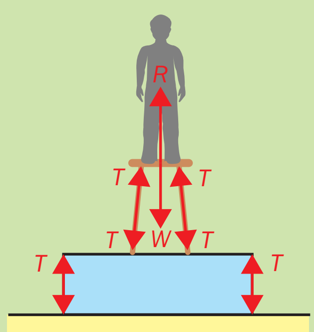
Tension (stretching tow bar) pulling car back, pulling trailer forward.

b) A pillar at the end of a wall consists of a concrete sphere on top of a concrete column.



Thrust acts upwards on sphere, weight acts downwards on column.

c) A child actor stands on a stool which is placed on a rostrum.



Reaction from stool acts on child.
Weight of child acts on stool.
Other forces acting include thrust from stool legs on rostrum and upwards force from rostrum on stool.

Practice

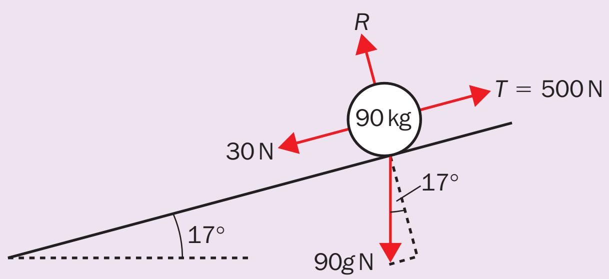
Throughout this exercise take $g = 10\ \text{ms}^{-2}$.

1 A man sits on a chair. Will the force in the chair legs be tension or thrust?

2 Find the acceleration of a particle of mass 0.05 kg, if acted upon by a force of 52 N.

3 What force will cause an object of mass 125 kg to accelerate at $0.4\ \text{ms}^{-2}$?

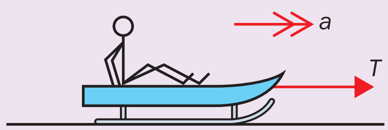
4 A body is acted upon by forces as shown in the diagram below.
What will be its acceleration parallel to the plane?



5 A child on a sledge is being pulled along by a horizontal rope, against resistance forces of 180 N. The combined mass of the child and sled is 62 kg. Show all this information on a sketch.

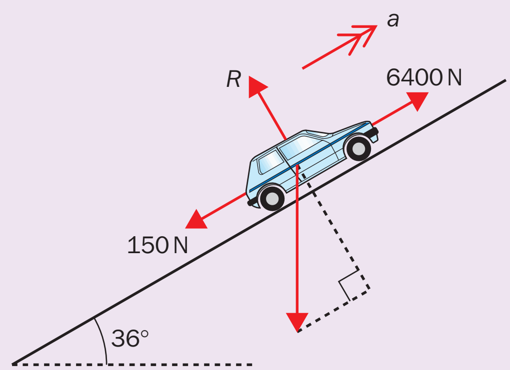
a) If the sledge moves at a steady speed, state the magnitude of the tension in the rope.

b) If the sledge accelerates at $0.25\ \text{ms}^{-2}$, find the new tension in the rope.



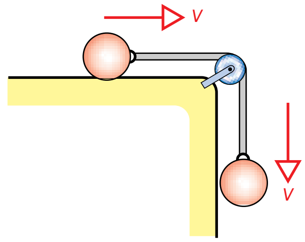
6 A boy on rollerskates is holding a javelin. How will Newton's Third Law apply to this situation if he tries to throw the javelin. How could he succeed in throwing the javelin without moving himself?

7 If the car of mass 1 tonne shown in the diagram is producing a tractive force of 6400 N, find the value of the normal reaction force acting from the plane on the car and calculate the acceleration of the car.



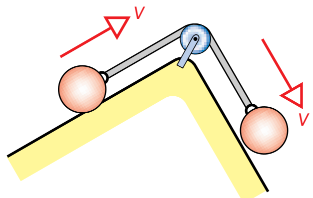
Connected particles

What will be the main feature of the motion of two particles that are connected by a string?
Providing the string (we use this word to mean any kind of cable or rope) remains taut, both particles will move the same **distance**, they will travel at the same **speed** and have the same **acceleration**.



Why will they not necessarily have the same *velocity*?
Depending on the situation, they may not both move in the same *direction*, even though they will have the same *speed*.

The main fact that will make the mathematical solution possible is that the tension in the string affecting both will have the same magnitude.
This is an example of Newton's Third Law in action.



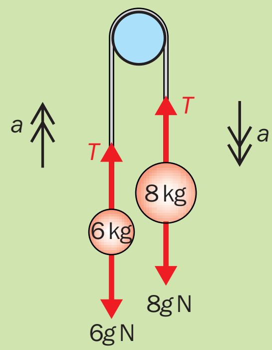
Since we are considering the forces acting on both particles, we will have to use an equation that links forces and acceleration. Which is that?
It will be Newton's Second Law in the form: $\mathbf{F} = m\mathbf{a}$.

The standard method we will use is:

- draw a force diagram showing all relevant forces,
- label the acceleration on each of the connected particles,
- write an **equation of motion** for each particle, using $\mathbf{F} = m\mathbf{a}$,
- add the equations (to eliminate the tension) and find the acceleration,
- if required, substitute back into either equation to find the tension.

Example 9

Two particles, of masses 6 kg and 8 kg, are connected by a light, inelastic string passing over a smooth fixed peg. Find the acceleration of the system when it is released from rest and the tension in the string. (Let $g = 9.8\text{ m s}^{-2}$.)



First, we draw the force diagram.

A peg, like a pulley, is a support from which the system is suspended.
The size of the tension in the string is the same at either end, and acts to pull the ends of the string towards the centre.
Both particles have the same acceleration.

We use $\mathbf{F} = m\mathbf{a}$, where $\mathbf{F}$ is the resultant force acting in the direction of the acceleration.

For the 8 kg mass: $8g - T = 8a$ ....(1)

For the 6 kg mass: $T - 6g = 6a$ ....(2)

Adding (1) + (2):

$$8g - T + T - 6g = 8a + 6a$$
$$2g = 14a$$
$$\frac{2g}{14} = a$$
$$\therefore \quad a = \underline{1.4\text{ m s}^{-2}}$$

Equation (2) $\Rightarrow$

$$T = 6a + 6g$$
$$T = 6 \times 1.4 + 6 \times 9.8$$
$$\therefore \quad T = \underline{67.2\text{ N}}$$

In example 9, it was obvious which way the system would move, once released; the heavier side would accelerate downwards.

What if it's impossible to tell just by looking?
Label the acceleration in the most likely direction and use the same method.
If the direction was wrong, the acceleration will come out negative.
You will then know that the system moves with this acceleration in the *opposite* direction.

Notice that we only consider the forces acting on one particle at a time.
The connection comes when we add the equations of motion, *not* when we write them.

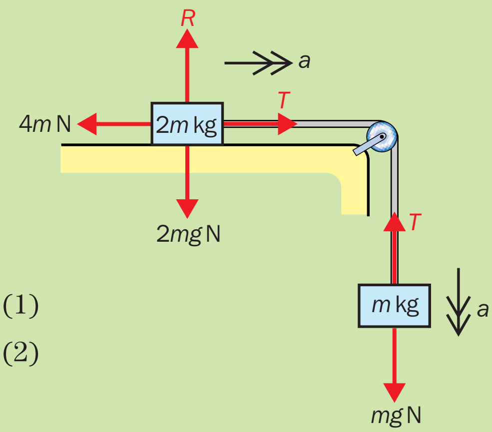
We only use $\mathbf{F} = m\mathbf{a}$ for forces parallel to the direction of the direction of motion.
Any forces perpendicular to the motion must balance out.

Example 10

A solid block, of mass $2m$ kg, is resting on a horizontal table. It is connected by a light, inelastic string passing over a smooth fixed pulley to a second block of mass m kg. When the system is released from rest the block on the table experiences a frictional force of $4m$ N.

a) Find the magnitude of the normal contact force acting on the block on the table.

b) Find the acceleration of the system when it is released from rest, and the tension in the string. (Let $g = 10\,\text{m s}^{-2}$.)



Again, we draw the force diagram showing the tension in the string and the same acceleration for each block:

a) The vertical forces on the block on the table must balance, therefore: $R = \underline{2mg\text{ N}}$

b) We use $\mathbf{F} = m\mathbf{a}$:

For the hanging mass: $mg - T = ma$ ....(1)

For the mass on the table: $T - 4m = 2ma$ ....(2)

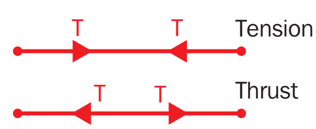
Adding (1) + (2):

$$mg - T + T - 4m = ma + 2ma$$
$$10m - 4m = 3ma$$
$$6m = 3ma$$
$$6 = 3a$$
$$2 = a$$
$$\therefore \quad a = \underline{2\text{ m s}^{-2}}$$

Equation (2) $\Rightarrow$

$$T = 2ma + 4m$$
$$T = 2m \times 2 + 4m$$
$$\therefore \quad T = \underline{8m\text{ N}}$$

What if the particles are connected by a rod that is in thrust, rather than in tension? What would be an example of that?
A good example would be the tow-bar between a car and a trailer.
For **tension**, the forces act towards the centre, **pulling the ends in**.
For **thrust**, the forces act away from the centre, **pushing the ends out**.



Maths in Action: A Comfortable Ride

One of the biggest changes in city architecture in recent years is that buildings have been getting dramatically taller. Skyscrapers with over 50 storeys are now commonplace, and a few now even reach to over 100 floors.

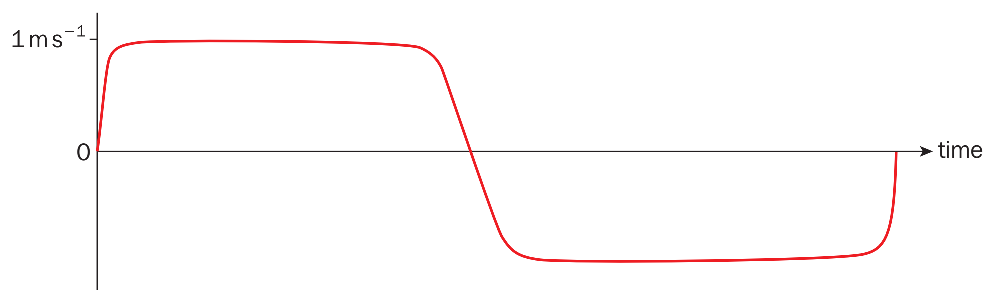
However, there are mathematical reasons why it is unlikely that many buildings will ever be much higher than this. One of the most important constraints on the height of a building comes from a factor you might not have predicted: the acceleration of the elevators.

A 100-floor skyscraper is of no use unless it is possible for people to conveniently and safely travel between the top and the ground. To deal with the large people movements in a skyscraper, the elevators need to travel quickly. It is no problem in theory for an elevator to travel at high speeds, such as 50 miles per hour or more. This would be no more uncomfortable than travelling in a car at that speed. Unfortunately, in order to get to those high speeds, the elevator needs to accelerate, and that is where the problems begin.

PICTURE TO BE INSERTED
TO BE SUPPLIED

You have probably experienced the sensation of your stomach lurching towards your throat as a lift plunged downwards. This sensation is caused by the lift's acceleration, which creates a sense of partial weightlessness. If the lift was allowed to drop in free-fall, you would experience complete weightlessness, and could float around the lift capsule. Joyriders would find this exciting, but for everyone else it would lead to nausea. In fact, to ensure passengers will be comfortable (and won't for example smear lipstick if they try to apply it while in motion!), lift designers normally restrict the acceleration upwards or downwards to 10% of gravitational acceleration, g, or about 1 ms^{-2}.

A lift that spends its entire journey accelerating and then decelerating at this maximum level of 1 ms^{-2} has this graph.



So how fast might the elevator travel, and how long will it take to climb to the top of a 100 storey building? If we assume the distance between floors is 4 metres making the building 400 metres high, we can work out the time from the standard formula: $s = ut + \frac{1}{2}at^2$.

For the first half of the journey, initial speed is zero and the elevator accelerates at 1 ms^{-2}:

$200 = 0 + 0.5 \times 1 \times t^2$, so $t = \sqrt{400} = 20$ seconds.

This halfway level is the point where the elevator reaches maximum speed.

We can work out the peak elevator speed using the formula:

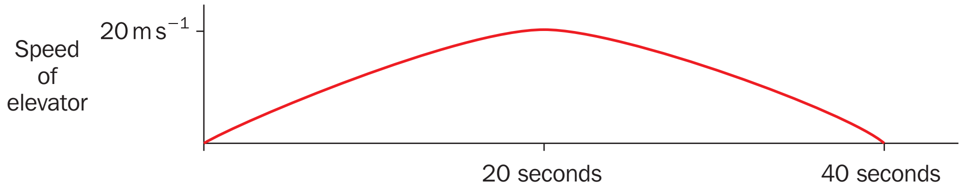
$v = u + at$

u (initial speed) is zero, so:

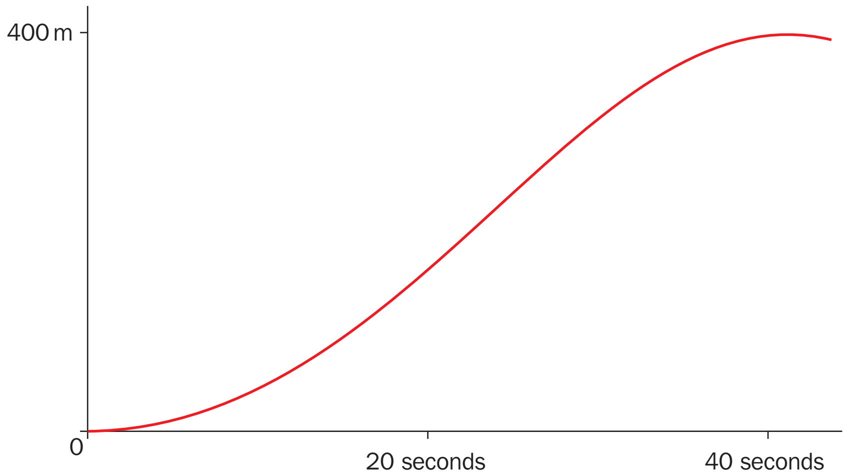
Peak speed $= 0 + (1 \times 20) = 20\ \text{ms}^{-1}$.

The second half of the journey is a mirror image of the first half, and will also take 20 seconds, making the total journey to the top 40 seconds.

The graph of the elevator's speed will look like this.



And its height plotted against time will look like this:



By these crude calculations it will take a lift almost a minute and a half to do a return journey from the ground to the top floor in a 100 storey skyscraper, and that is ignoring loading times and assuming it doesn't have to stop to drop off passengers on the way.

A single lift might be able to carry 50 people at most. At peak times of the morning, there will be hundreds of people arriving at the building every minute and to prevent queues building there must be enough lifts to ferry them all. But the more lift shafts there are, the less space there is for desks and people. There comes a stage where in order to handle the vast numbers of people moving up and down a tall building, the entire floor space needs to be taken up by lift shafts!

This great elevator conundrum explains why skyscrapers of 200 floors or more are unlikely to be viable in the years to come.

Summary

• Acceleration due to gravity:	$g = 9.8\ \text{ms}^{-2}$	(near the Earth's surface)
• Weight force:	mg N	(where m is the mass in kg)
• Newton's First Law	A body will continue to remain at rest or move at constant speed in a straight line unless an external force makes it act otherwise.	
• Newton's Second Law	A resultant force acting on a body produces an acceleration which is proportional to the resultant force.	
• Newton's Second Law (in equation form)	$\mathbf{F} = m\mathbf{a}$.	
• Newton's Third Law	For every action there is an equal and opposite reaction.	

End of Chapter Questions

1 Can you complete these sentences?

a) Gravity acts towards …

b) The force produced when an object touches a surface is …

c) The weight of an object is its … multiplied by …

d) Resistance forces act to oppose …

e) For motion in a straight line at constant speed, the forces must …

f) The simple equation for Newton's Second Law is …

g) For motion on a slope we use $F = ma$ … to the slope.

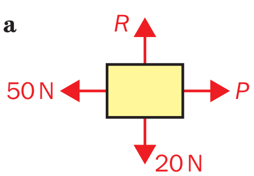
h) A simple statement for Newton's Third Law is 'Forces come in …'.

2 What is the weight of a beetle of mass 5.1 g? (Take $g = 10\ \text{ms}^{-2}$.)

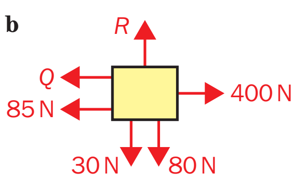
3 A body of mass 54 kg lies on a plane inclined at 39° to the horizontal. Express the weight of the body in the form $a\mathbf{i} + b\mathbf{j}$, where $\mathbf{i}$ acts parallel and down the plane and $\mathbf{j}$ is perpendicular to the plane. (Take $g = 9.8\ \text{ms}^{-2}$.)

4 If each of the bodies shown is travelling with constant speed, find each of the unknown forces.

a



b



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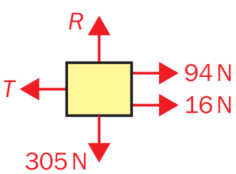
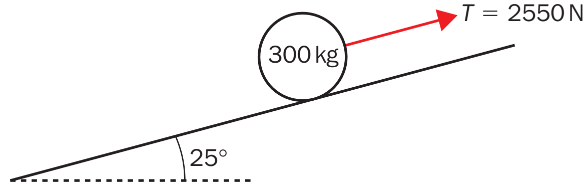


5 Find the acceleration up the plane of the body in the diagram. (Take $g = 9.8\ \text{ms}^{-2}$.)



6 At time $t = 0$, a force of magnitude 12 N is applied to a particle of mass 6 kg which is at rest.

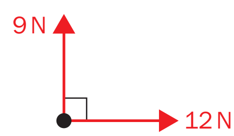
a) Find the acceleration of the particle.

b) Find the speed of the particle when $t = 3$.

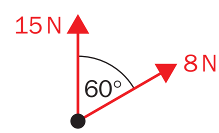
7 A particle of mass 400 g is moving along a straight line at $12\ \text{m s}^{-1}$. The speed of the particle is reduced to $4\ \text{m s}^{-1}$ by the action of a constant resultant force of magnitude 8 N. Find the length of time for which the force acts on the particle.

8 Each diagram shows the forces acting on a particle of mass 5 kg. Find, to 3 s.f. where appropriate, the acceleration of the particle.

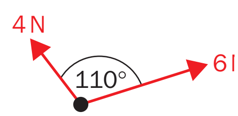
a)



b)



c)



9 A resultant force $(6\mathbf{i} - 12\mathbf{j})$ N acts on a body of mass 3 kg.

a) Find the acceleration of the particle in the form $a\mathbf{i} + b\mathbf{j}$.

b) Find the magnitude of the acceleration.

10 A particle of mass 6 kg is moving with an initial velocity of $(2\mathbf{i} + \mathbf{j})\ \mathrm{m\,s^{-1}}$. It is then acted upon by a resultant force $(18\mathbf{i} - 12\mathbf{j})$ N.

a) Find the acceleration of the particle.

b) Find the speed of the particle after 4 seconds.

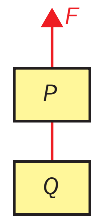
11 A particle of mass 200 g is acted upon by forces $(a\mathbf{i} + 3\mathbf{j})$ N, $(-\mathbf{i} - 17\mathbf{j})$ N and $(12\mathbf{i} + b\mathbf{j})$ N. Given that the particle accelerates at $2.5\ \mathrm{m\,s^{-2}}$ in the direction of the vector $(3\mathbf{i} - 4\mathbf{j})$, find the values of a and b.

12 A suitcase of mass 10 kg is pulled along a smooth horizontal surface by a horizontal rope. The suitcase accelerates at $0.2\ \mathrm{m\,s^{-2}}$. Find the tension in the rope.

13 A box of mass 20 kg is dragged along a rough horizontal plane at constant speed by a force of magnitude 60 N which is inclined at an angle of 60° to the horizontal. Find the friction force acting on the box.

14 A particle of mass 0.25 kg is placed on a smooth plane inclined at an angle of 30° to the horizontal and released from rest. Find its acceleration.

15 Particle P of mass 3 kg is suspended by a light inextensible string. Particle Q of mass 2 kg is attached to P by means of a second light inextensible string. A force of magnitude F is applied vertically upwards to the upper string and causes the particles to accelerate upwards at $2.2\ \mathrm{m\,s^{-2}}$. Find



a) the magnitude of F,

b) the tension in the string which joins the two particles.

16 A lorry of mass 1200 kg tows a trailer of mass 800 kg by means of a light tow-bar along a straight horizontal road. The lorry's engine produces a driving force of magnitude 10 000 N and the lorry and the trailer experience resistance forces of magnitude 2400 N and 1600 N respectively.

a) Find the acceleration of the system.

b) Find the tension in the tow-bar between the lorry and the trailer.

The lorry then switches off its engine. Given that the resistance forces remain unchanged, find

c) the deceleration of the system,

d) the force in the tow-bar.

How to make the Examiner happy

- Draw a BIG, clear force diagram and show on it ALL the information from the question. Before solving the problem, read the question through to make sure you have it all written down correctly.
- Indicate the angle for any inclined forces.
- Use the distinctive arrow-head notation in this text book to differentiate the various mechanical quantities on your diagram.
- Make it clear whether you are using Newton's Second Law in scalar form or vector form.
- Where appropriate give answers in exact form (i.e. including surds or g) before calculating a numerical value. Use these exact forms in subsequent calculations, or make efficient use of your calculator's memory functions.
- Give numerical answers to 3 s.f. and angles to 1 d.p. unless there is a good reason to do otherwise (i.e. the question says so!).

6 Momentum and Impulse

Have you met the word ‘**momentum**’ in connection with a moving object? We sometimes say that it is difficult to stop a moving object because of its momentum. Can an object at rest have ‘momentum’?

How can we measure the momentum of an object or vehicle?

We also talk about ‘acting on impulse’ when we mean doing something suddenly, on the spur of the moment.
The word ‘**impulse**’ also has a specific meaning in mechanics.

In this chapter you will learn:
- how mathematicians define the terms ‘momentum’ and ‘impulse’,
- how to calculate the momentum of a moving body,
- how to use the relationship between impulse and momentum,
- why momentum is conserved in collisions,
- what happens when particles ‘coalesce’,
- how to apply the Principle of Conservation of Linear Momentum.

Momentum

TAKE IN PHOTO P6.1
table tennis ball being hit hard
TO BE SUPPLIED

If momentum is a property of moving objects, what factors affect its ‘size’ or magnitude? What gives one object more momentum than another?

We can begin to answer this question by taking the commonsense view that ‘momentum is what makes an object keep moving’.
Think about two balls rolling towards you, both at the same speed; one is a football and the other is a bowling ball.
Which is harder to stop?
Instinct tells us that the heavy bowling ball will be more difficult to stop.
Greater mass means more momentum if the speed is the same.

Now imagine catching a ping-pong ball.
Will it be harder to stop when it is thrown gently to you, or when it is hit at top speed towards you, straight from the bat?
The faster-moving ball would have greater momentum, although the masses are identical.

In fact, the only two factors that matter are **mass** and **velocity**.

The definition of momentum:

$$\textbf{momentum} = \textbf{mass} \times \textbf{velocity}$$

or $$\textbf{momentum} = mv$$

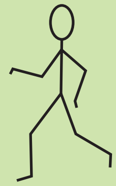
The units of momentum will be [mass] × [velocity] = $\mathrm{kg\,m\,s^{-1}}$.
Remember the units of force are Newtons, and $\mathrm{N = kg\,m\,s^{-2}}$.
These units are very much like the ones we require, in fact:

$$\begin{aligned} \mathrm{N \times s} &= \mathrm{kg\,m\,s^{-2} \times s} \\ &= \mathrm{kg\,m\,s^{-1}}. \end{aligned}$$

The units of momentum are therefore **N s**, ‘newton seconds’.

Example 1

A man of mass 80 kg is walking at $2.5\ \text{m s}^{-1}$. What is his momentum?

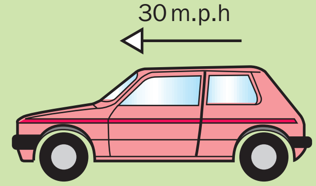


$$\begin{aligned}\text{Momentum} &= mv\\ &= 80 \times 2.5\\ &= \underline{200\ \text{N s}}\end{aligned}$$

We need to make sure we use SI units for all calculations.
If any information is given in different units, first of all we need to convert it to SI units.

Example 2

What is the momentum of a car of mass 0.81 tonnes, travelling at 30 m.p.h.?



$$\begin{aligned}0.81\ \text{tonnes} &= 0.81 \times 1000\ \text{kg}\\ &= 810\ \text{kg}\\ 30\ \text{miles per hour} &= 30 \times 1.6\ \text{km per hour}\\ &= 48\ \text{km per hour}\\ &= 48\,000\ \text{metres per hour}\\ &= \frac{48\,000}{60 \times 60}\ \text{metres per second}\\ &= \frac{40}{3}\ \text{m s}^{-1}\\ \therefore \quad \text{Momentum} &= mv\\ &= 810 \times \frac{40}{3}\\ &= \underline{10\,800\ \text{N s}}\end{aligned}$$

Do you remember the difference between vector and scalar quantities?
Which kind is momentum?
Scalars have only a magnitude, while vectors have both size and direction.

Back p. 8

Momentum is a vector quantity:

Its direction is the same as the direction in which the body is moving.

If the velocity is given as a vector, we use the definition:

momentum $= m\mathbf{v}$

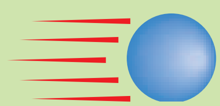
(A bold letter for velocity shows it is a vector.)

We know that multiples of a vector are parallel to each other. It therefore follows that the momentum will be in the same direction as the velocity.

Example 3

A particle of mass 0.75 kg is travelling with velocity $(-16\mathbf{i} + 28\mathbf{j})\ \text{m s}^{-1}$. Find its momentum.

$$\begin{aligned}\text{Momentum} &= m\mathbf{v}\\ &= 0.75 \times (-16\mathbf{i} + 28\mathbf{j})\\ &= 0.75 \times (-16)\mathbf{i} + 0.75 \times 28\mathbf{j}\\ &= \underline{(-12\mathbf{i} + 21\mathbf{j})\ \text{N s}}\end{aligned}$$



If the object is moving in only **one dimension** (along a straight line) then a **plus or minus** sign will be enough to indicate which way it is moving.

Impulse

In tennis, the direction and speed of the ball is changed by a very brief contact with the racket. The same is generally true of all ball sports.

The effect of a force applied for a very short time, like this, is called an **impulse**.

We make the assumption that the magnitude of the force acting is constant, which seems reasonable as it acts for such a short time.

The definition of impulse:

impulse = force × time

or $J = Ft$ (J is used for the magnitude of an impulse)

or $\mathbf{J} = \mathbf{F}t$ (if $\mathbf{F}$ is given as a vector, $\mathbf{J}$ will be a vector)

When impulse is a vector, its direction is the same as the force involved.

What are the units of impulse?
Think about the quantities involved and work it out for yourself.

The units of impulse must be [force] × [time] = N s.
These are the same as the units of momentum.

Example 4

What is the magnitude of the impulse produced by a constant force of 2000 N acting for 0.0037 s?

$$\begin{aligned}\text{Impulse} &= F \times t \\ &= 2000 \times 0.0037 \\ &= \underline{7.4\ \text{N s}}\end{aligned}$$

In practical terms, which of the quantities impulse, force and time is it hardest to measure?
It is difficult to measure the very short time of application of the force.

How could you find out how long a tennis ball is in contact with the racket?
Perhaps it could be found from slow-motion film footage.
It could be easily calculated if we knew the force and the impulse.

Example 5

A particle receives an impulse of magnitude 11.6 N s as a result of a constant force of 250 N acting for a time t seconds. Find the value of t.

$$\begin{aligned}\text{Since} \qquad J &= Ft \\ 11.6 &= 250t \\ \frac{11.6}{250} &= t \\ \therefore \qquad t &= \underline{0.0464\ \text{s}}\end{aligned}$$

On the other hand, how do we measure impulse?
If their units are the same, what is the connection between impulse and momentum?

Remember first, that $F = ma$.
If the force is assumed to be constant, then the acceleration is also constant and we can use: $v = u + at$
Rewrite this to make a the subject of the formula:

$$v - u = at$$

$$\frac{v - u}{t} = a$$

Now, since

$$\begin{aligned} \text{impulse} &= F \times t \\ &= ma \times t \qquad \text{(replacing } F \text{ with } ma\text{)} \\ &= m\frac{v-u}{t} \times t \qquad \left(\text{replacing } a \text{ with } \frac{v-u}{t}\right) \\ &= m\,(v - u) \\ \text{impulse} &= mv - mu \end{aligned}$$

If u and v represent initial and final velocity, what does this equation mean?
We can call mv the **final momentum** and mu the **initial momentum**.

The impulse–momentum principle:

impulse = change in momentum

or **impulse = final momentum – initial momentum**

or $J = mv - mu$

Therefore the impulse can be calculated from the initial momentum and the final momentum. These are relatively easy to measure as they depend only on mass and speed.

Example 6

A tennis ball of mass 50 g is rolling along the ground at $0.8\ \text{m s}^{-1}$, when it is tapped from behind by a racket moving in the same direction. Immediately afterwards its speed is $1.4\ \text{m s}^{-1}$.
What was the magnitude of the impulse applied to the tennis ball?

First, note that the mass of the ball in SI units is 0.05 kg.

Since

$$\begin{aligned} J &= mv - mu \\ J &= 0.05 \times 1.4 - 0.05 \times 0.8 \\ &= 0.07 - 0.04 \\ &= \underline{0.03\ \text{N s}} \end{aligned}$$

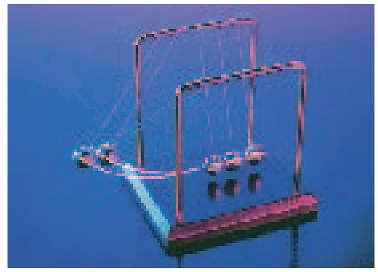
What assumptions or simplifications have been made here?

- The ball has been treated as a particle.
- All the motion happens in a straight line.
- The effects of air resistance, friction and rolling have been ignored.

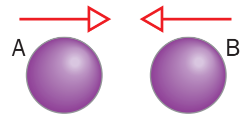
In particular, only the speeds **just before** and **just after** the impulse are used. The ball may have been slowing down to reach the initial speed of $0.8\ \text{m s}^{-1}$, and will slow down again from its new speed of $1.4\ \text{m s}^{-1}$.

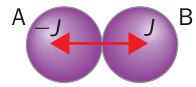
Conservation of linear momentum

Have you ever played with one of the ‘executive toys’ shown here?
What happens to momentum when two moving objects collide?

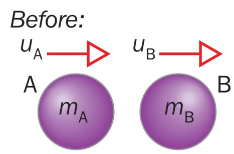


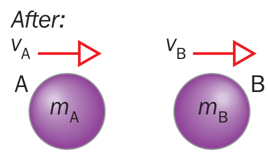
To answer this, let’s look first at the simple case of two moving spheres colliding head-on.
Where will there be any impacts or impulses?
The point of impact will be the point where the two spheres touch.
Each sphere will exert an impulse on the other.
Remember that an impulse is a force acting for a short time.
What does Newton’s Third Law tell us about forces?
‘For every action there is an equal and opposite reaction’,
that is: ‘***forces come in equal and opposite pairs***’.
Therefore the impulse that sphere A exerts on sphere B is the same in magnitude as the impulse that sphere B exerts on sphere A.
On the diagram, we can label them as equal and opposite using J and $-J$.





In the following steps, the letters used are defined as follows:
- m_A and m_B stand for the masses of sphere A and sphere B, respectively,
- u_A and u_B stand for the initial speeds of the two spheres,
- v_A and v_B stand for the final speeds of the two spheres.





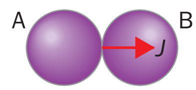
Next, we use

impulse = change in momentum

for each of the two impulses in turn.

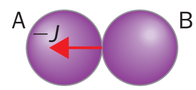
The impulse produced by sphere A affects the momentum of sphere B.

So: $J = m_B v_B - m_B u_B$ (1)



Similarly, the impulse produced by sphere B affects the momentum of sphere A.

$$-J = m_A v_A - m_A u_A \qquad (2)$$



As the two impulses are equal and opposite, we can add them together to get zero.
Adding equation (1) and equation (2):

$$0 = m_B v_B - m_B u_B + m_A v_A - m_A u_A$$

Moving the negative terms to the left-hand side:

$$m_B u_B + m_A u_A = m_B v_B + m_A v_A$$

Changing the order slightly:

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

In words this can be expressed as:

total momentum before collision = total momentum after collision

What does this equation tell us about collisions?
There is the same amount of momentum beforehand as there is afterwards.

The Principle of Conservation of Linear Momentum (CLM):

total momentum before collision = total momentum after collision

or $\boldsymbol{m_A u_A + m_B u_B = m_A v_A + m_B v_B}$

This equation will be true provided there are no external forces acting (such as friction or air resistance).

When we apply this equation to solve a problem, it is important to do two particular things:

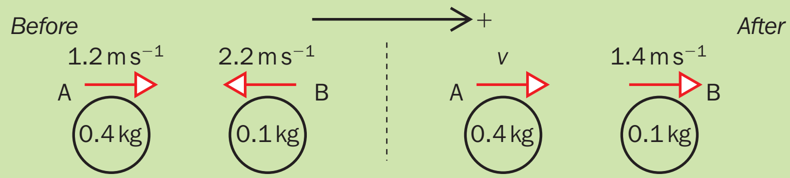
- always draw 'Before' and 'After' diagrams,
- label clearly the positive direction.

The positive direction is usually taken to be 'to the right', like the direction of the x axis. Any velocity in the opposite direction will have a negative sign and therefore the body in question would have a negative momentum.

Example 7

Two smooth spheres of equal radii, A and B, are sliding towards each other across a smooth horizontal surface. A has velocity $1.2\ \mathrm{m\,s^{-1}}$ and mass 0.4 kg. B has velocity $2.2\ \mathrm{m\,s^{-1}}$ and mass 0.1 kg. They collide directly and the direction of B is reversed and its velocity is reduced to $1.4\ \mathrm{m\,s^{-1}}$. What will be the new velocity of A? In which direction will it be travelling?

Draw 'before' and 'after' diagrams, with a clearly stated positive direction.
As we don't know yet which way A will end up going, give it a possible velocity in the positive direction.
(If this assumption is wrong, the answer will be negative and we will know it is going the other way.)



CLM: total momentum before collision = total momentum after collision

$$
\begin{aligned}
0.4 \times 1.2 + 0.1 \times (-2.2) &= 0.4 \times v + 0.1 \times 1.4 \\
0.48 - 0.22 &= 0.4v + 0.14 \\
0.26 &= 0.4v + 0.14 \\
0.12 &= 0.4v \\
0.12 \div 0.4 &= v \\
0.3 &= v
\end{aligned}
$$

The new speed of sphere A is $0.3\ \mathrm{m\,s^{-1}}$. Its direction is the same as it was before the collision.

It seems remarkable that the momentum is conserved in a case like this example, where both the spheres are moving *more slowly* at the end. The calculations show that the *total* of the positive and negative momentums is the same before and after the collision. Momentum has been conserved but *energy has been lost* from the system.

How many modelling assumptions can you pick out in the last question?

Typical modelling assumptions for two spheres colliding:

- smooth spheres
- equal radii
- sliding
- smooth horizontal surface
- collide directly (head-on)
- no other external forces acting

Collisions and explosions

We can apply the principle of Conservation of Linear Momentum (CLM) to a variety of different situations. We have already seen how to deal with two spheres colliding directly and separating again.
What other kinds of situations involving collision or impulse are there?

There are three other possibilities that need considering:
- colliding with a stationary object,
- two objects which coalesce,
- explosive separation.

TAKE IN PHOTO P6.6
2 cars, one driven into
back of the other
TO BE SUPPLIED

Colliding with a stationary object

It sometimes feels odd using the same method that works for two moving objects in the case where one body strikes an unmoving (stationary) object.

What are the similarities and differences?

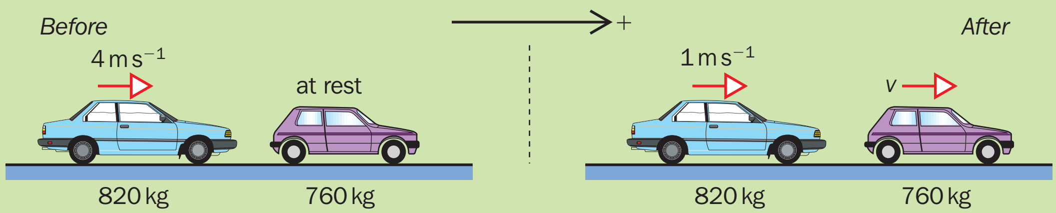
Both objects exert an impulse on each other. However, the stationary object has no momentum to start with and after the collision can only move in the direction of the impulse it has received.

Example 8

A car of mass 760 kg is stationary when it is 'shunted' from behind by a car of mass 820 kg, travelling at 4 m s^{-1}. The speed of the moving car is reduced to 1 m s^{-1} by the impact.

What is the speed of the other car immediately after the impact?

As usual, we can take the positive direction to the right.
All the motion is in this direction in this question.



CLM: total momentum before collision = total momentum after collision

$$820 \times 4 + 760 \times 0 = 820 \times 1 + 760 \times v$$
$$3280 = 820 + 760v$$
$$3280 - 820 = 760v$$
$$2460 = 760v$$
$$2460 \div 760 = v$$
$$3.24 \text{ (3 s.f.)} = v$$

Immediately after being hit from behind, the other car moves forward with velocity 3.24 m s^{-1} (3 s.f.).

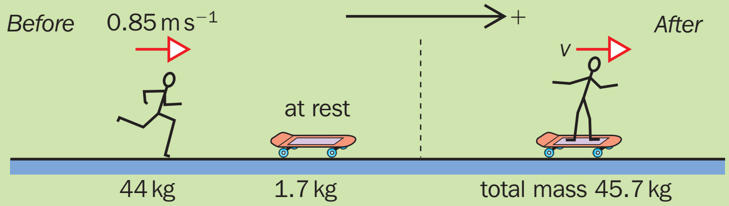
Objects which coalesce

'Coalescing' means that the two objects join together and move as though they were one single object. Can you think of some examples?
How about these: train trucks in the shunting yard, two space craft docking, a bullet impacting into a charging tiger, a fly hitting the windscreen of a car.

CLM still applies, but the single body after the collision has a mass equal to the sum of the masses before the collision.

Example 9

A boy of mass 44 kg is travelling with a horizontal velocity of $0.85\ \text{m s}^{-1}$ as he lands on his stationary skateboard. If the mass of the skateboard is 1.7 kg, what will be the velocity of the boy and the skateboard immediately after they begin to move together?



CLM: total momentum before = total momentum after

$$44 \times 0.85 + 1.7 \times 0 = (44 + 1.7) \times v$$
$$37.4 + 0 = 45.7v$$
$$37.4 = 45.7v$$
$$37.4 \div 45.7 = v$$
$$v = 0.818 \text{ (3 s.f.)}$$

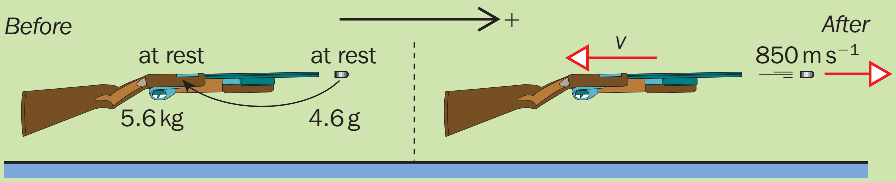
Immediately after landing, the boy and skateboard move off with a velocity of $0.818\ \text{m s}^{-1}$ (3 s.f.).

Explosive separation

This sounds quite dramatic, but in this context means the sudden powerful separation of two objects. (A single object exploding into lots of little pieces might be a bit too complicated to model at this stage in the course!)
A good example would be the firing of a bullet from a rifle.
We need to remember that the rifle is affected by the shot as much as the bullet. The person firing the rifle would feel the recoil of the shot.

Example 10

A rifle with mass 5.6 kg fires a bullet of mass 4.6 g with an estimated muzzle velocity of $850\ \text{m s}^{-1}$.
With what velocity will the rifle recoil immediately after the shot is fired?



CLM: total momentum before = total momentum after

$$5.6 \times 0 + 0.0046 \times 0 = 5.6 \times (-v) + 0.0046 \times 850$$
$$0 = -5.6v + 3.91$$
$$5.6v = 3.91$$
$$v = 3.91 \div 5.6$$
$$v = 0.698 \text{ (3 s.f.)}$$

The rifle recoils with a velocity of $0.698\ \text{m s}^{-1}$ (3 s.f.) immediately after the shot is fired.

Summary

- **Momentum** = mass × velocity — momentum = $m\boldsymbol{v}$
- **Impulse** = Force × time — $\boldsymbol{J} = \boldsymbol{F} \times t$
- **Impulse = change in momentum** — $\mathbf{J} = m\mathbf{v} - m\mathbf{u}$
- The units of momentum and impulse are **N s** (newton seconds).
- The Principle of Conservation of Linear Momentum (CLM):

  total momentum before collision = total momentum after collision

End of Chapter Questions

1 Can you complete these sentences?

a) The units of momentum are …

b) The result of a force acting for a short time is called an …

c) Momentum is the product of … and …

d) The force which produces an impulse is assumed to be …

e) Linear momentum is conserved provided there are no …

f) A head-on collision is described as 'colliding …' .

2 What word is used to describe the situation where two particles collide and remain attached after the collision?

3 What is the momentum of a train of mass 24 tonnes, travelling at $0.14\ \text{m s}^{-1}$?

4 Work out the momentum of a fly of mass 0.068 g, flying at $1.2\ \text{m s}^{-1}$.

5 Calculate the magnitude of the impulse of a constant force of 1500 N acting for 0.009 seconds.

6 A ping-pong ball of mass 2 g has an initial speed of $1.85\ \text{m s}^{-1}$, but after being hit by the player's bat it has a new speed of $3.42\ \text{m s}^{-1}$ in the opposite direction.

a) Calculate the magnitude of the impulse given to the ping-pong ball.

b) If the bat is in contact with the ball for 0.088 s, find also the magnitude of the constant force acting to produce the impulse.

7 Two smooth spheres A and B, of equal radii, are sliding towards each other across a smooth horizontal surface. A has velocity $2.8\ \text{m s}^{-1}$ and mass 0.5 kg. B has velocity $1.2\ \text{m s}^{-1}$ and mass 0.8 kg.

After the collision, A is brought to rest. What will be the new velocity of B?

8 A youth of mass 66 kg jumps with velocity $0.92\ \text{m s}^{-1}$ onto a stationary supermarket trolley of mass 26 kg. What will be the velocity with which the youth riding the trolley moves off?

9 exam — A ball of mass 500 g is moving at $4\ \text{m s}^{-1}$ when it receives an impulse of magnitude 0.8 Ns in the opposite direction to its motion. Find the speed of the ball immediately after the impulse.

10 exam — A particle P of mass 2 kg is dropped from a height of 0.6 m onto horizontal ground.

a) Find, to 2 s.f., the speed of P just before it hits the ground.

The particle rebounds vertically from the ground with speed $2.5\ \text{m s}^{-1}$.

b) Find, to 2 s.f., the magnitude of the impulse exerted on the ground by the ball in the impact.

11 exam — Two particles P and Q have masses 0.5 kg and 0.2 kg respectively. They are moving in the same direction along the same straight line with speeds $5\ \text{m s}^{-1}$ and $2\ \text{m s}^{-1}$ respectively when they collide. After the collision, P continues to move along the same line with speed $4\ \text{m s}^{-1}$. Find the speed of Q after the collision.

12 exam — A railway truck of mass 35 tonnes is moving with a speed of $4\ \text{m s}^{-1}$ when it collides with a stationary truck of mass 40 tonnes. The trucks couple together and move off.

a) Find the common speed of the trucks after the collision.

b) Find the magnitude of the impulse between the trucks during the collision.

13 Two particles A and B are at rest on a smooth horizontal plane. The particles are connected by a light inextensible string which is initially slack. Particle A is projected away from particle B with a speed of $3\ \text{m s}^{-1}$. The mass of A is 3 kg and the mass of B is 2 kg.

a) Find the speed of the particles just after the string goes taut.

b) Find the magnitude of the impulse transmitted through the string as the string goes taut.

14 A bullet of mass 20 g is fired horizontally into a wall with a speed of $500\ \text{m s}^{-1}$. The bullet becomes embedded in the wall, coming to rest in 0.01 seconds.

a) Find the magnitude of the impulse exerted on the wall by the bullet.

b) Find the magnitude of the constant resistive force exerted on the bullet by the wall.

15 A heavy metal brick of mass 4 kg is dropped from a height of 3 m above soft horizontal ground. On striking the ground it sinks in a distance of 2 cm before coming to rest. Find, to 3 s.f.,

a) the magnitude of the impulse exerted on the brick by the ground,

b) the magnitude of the constant resistive force exerted on the brick by the ground.

16 Particles P and Q are moving in opposite directions on a smooth horizontal table. Particle P has mass m and speed $3u$ and particle Q has mass km, where k is a constant, and speed u. The particles collide and as a result of the collision the direction of motion of each particle is reversed and the speed of each particle is halved.

a) Find the value of k.

a) Find, in terms of m and u, the magnitude of the impulse exerted on P in the collision.

17 A toy rocket of mass 150 g is designed to split into two parts, the smaller of which has mass 50 g. When the toy is moving with speed $4\ \text{m s}^{-1}$ it splits, with the larger part continuing to move in the same direction and the smaller part moving in the opposite direction. Given that the speed of the smaller part, $u\ \text{m s}^{-1}$, is half of the speed of the larger part, find the value of u.

How to make the Examiner happy

- Always draw a big, clear 'before' and 'after' diagram, showing clearly the positive direction you are choosing. This is best taken towards the right, as that is what we are used to from coordinate graphs.
- Include all the velocities and masses, in kilograms, on this diagram so that you have all the information in front of you.
- Remember that objects moving to the left will have a negative momentum.
- When using CLM, always write a momentum equation, even if you can see that the masses might cancel out (for example, where the collision is between two spheres, both of mass M kg).
- You might be asked about the assumptions that are commonly made when two spheres collide. We normally assume that the spheres have equal radii, move freely on a smooth surface without rolling and collide directly.

7 Friction

Although friction has already been mentioned in this book, we have not so far looked at its properties. It is a **force** with certain unique qualities.
Try to describe friction for yourself.

TAKE IN A/W A7.1
sandpaper rubbing wood
TO BE SUPPLIED

Where does friction happen?
It acts between the surfaces of two objects in contact.

What does it do?
It makes it difficult for smooth sliding to happen and may prevent movement altogether.
It makes it possible to grip and turn objects.

Friction is vital to many of the everyday activities of life.
Without it we couldn't walk, ladders would always fall down and no one would be able to twist the lid off the marmalade!
All doors would have handles because it would be impossible to grip and turn a doorknob!
Icy days and oily surfaces give us some idea of what it would be like without friction.

TAKE IN PHOTO
P7.1
car at skid pan
TO BE SUPPLIED

A car on a skid pan slides over the slippery surface

In this chapter you will learn:

- what causes friction,
- the direction in which friction acts,
- how friction is related to the normal reaction force,
- why friction can change magnitude,
- the formula for modelling friction,
- how to use or find the coefficient of friction (μ),
- how to solve problems which involve friction.

Defining friction

Even the smoothest of surfaces is not completely smooth at a microscopic level. Seen at great magnification, a smooth object sliding over a smooth surface may not look quite the same!

TAKE IN A/W A7.2
TO BE SUPPLIED

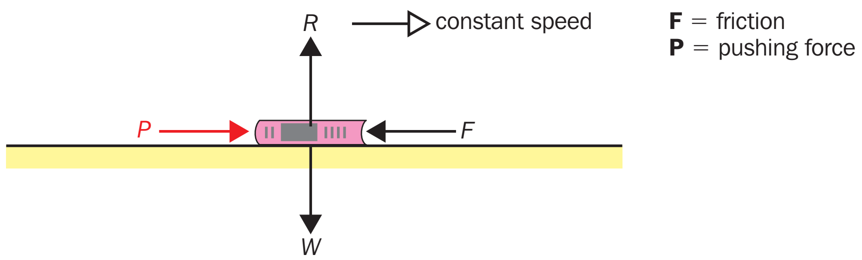
We know that friction makes it difficult for sliding to take place, but what features of friction can we model mathematically?
Let's begin by considering a situation where friction is likely to act.

Try doing this for yourself.
Place a book on a horizontal table, at the left hand side.
Push it from the left side to the right side of the desk.
Use as little force as possible and move it at a steady speed.
How was friction acting in this case?

TAKE IN A/W A7.3
hand pushing book
TO BE SUPPLIED

The first thing you probably noticed was that you had to increase the force you applied until the book started to move.
This is because friction was preventing the book from sliding until you applied enough force.
In which direction did the friction act?
It was hindering the motion of the book and so it was in exactly the opposite direction.

Were you able to move the book at a constant speed?
What does this tell us about friction?
Suppose we draw a force diagram of the situation once the book is moving at a steady speed across the table.



The book is not moving up or down in a vertical direction.
What does this tell us about **R** and **W**?
They must be equal and opposite.

If the speed is steady, the acceleration is zero.
What can we deduce about **P** and **F**?
By Newton's First Law, they too must be equal and opposite.

Did you feel you were exerting a constant force on the book?
This must mean that the magnitude of the friction was also constant.

Now try this experiment again with a pile of three books.
What changes do you notice?
It takes a greater force to get the books moving in the first place and to keep them going at a steady speed.
This suggests that the frictional force is greater than before.

In fact, the frictional force between the two surfaces is increased because the extra weight acting downwards presses the surfaces together more.

Here is a summary of the properties of friction that we have seen so far.

General principles of friction:

- Friction acts between two surfaces in contact.
- Friction always opposes motion (i.e. acts in the opposite direction).
- Friction is proportional to the force pressing the two surfaces together.
- There is no friction acting unless a force is applied.
- Friction only needs to be great enough to prevent motion if a small force is applied.

TAKE IN A/W A7.4
hand pushing books
TO BE SUPPLIED

The coefficient of friction

The 18th-century French physicist and military engineer, Charles Augustin de Coulomb, whose name is best known for his work on electricity, also developed the principles of friction into the model we use today.
Here are some definitions of important phrases related to friction.

Smooth contact means no friction is acting.

Rough contact means friction is likely to be acting.

Friction is a **resistance force** and only starts to come into play when there is a force attempting to cause motion. Friction will increase in value to prevent motion until it reaches its maximum value, written as $\mathbf{F_{max}}$.

When motion is *about to take place*, then the situation is in **limiting equilibrium** and any increase in the acting force will be sufficient to overcome the maximum value of friction and sliding will begin.

The maximum value of friction is proportional to the normal contact force acting. The coefficient of proportionality is called the **coefficient of friction, $\boldsymbol{\mu}$.** (This is the Greek letter for m and is called 'mu'.)

Charles Augustin de Coulomb, 1736–1806

Friction can take the following range of values:

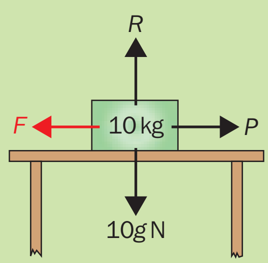
$0 \leqslant F \leqslant F_{\mathbf{max}}$, where $F_{\mathbf{max}} = \mu R$ (R is the normal contact force)

Friction only takes its maximum value, i.e. $F = F_{max}$, when the system is in limiting equilibrium or motion is taking place.

If the system is not in motion, friction only takes a large enough value to prevent motion happening.

Example 1

A block of wood with mass 10 kg is in rough contact with a wooden table.
The coefficient of friction between the two surfaces is 0.3.
A force, of magnitude P, is applied to the block. What will happen if a) $P = 25$ N, b) $P = 30$ N, c) $P = 50$ N?
State the magnitude of the frictional force and whether or not the block will move. (Take $g = 10\text{ m s}^{-2}$.)



First we should calculate R, and then the maximum value of F.

Resolving vertically: $R = 10\,g = 10 \times 10 = 100$ N

Since $F_{max} = \mu R$, $F_{max} = 0.3 \times 100 = 30$ N

As $F \leqslant F_{max}$, $F \leqslant 30$ N

a) If $P = 25$ N, F will take the same value as P, $F = 25$ N. The block will not move.

b) If $P = 30$ N, F will take its maximum value, $F_{max} = 30$ N. The block is now in limiting equilibrium.

c) If $P = 50$ N, F cannot exceed $F_{max} = 30$ N. The block will now move in the direction of the force P.

In example 1, could the magnitude of F ever exceed the value of P?
No. If P is small, F will be equal to P.
Friction cannot produce motion, which is what would happen if $F > P$.

What can we say about the value of μ if two surfaces are in smooth contact?
For smooth contact, $\mu = 0$ and therefore $F = 0$.

What is a typical value of μ?
For smooth but unpolished objects on a table, μ could be around 0.2 or 0.3.
For rubber tyres on a good road surface, μ might be 0.7 or 0.8.
On an icy surface we might have values of μ below 0.1.

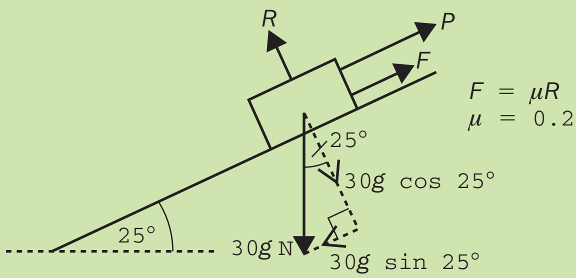
Friction on this surface will be low

Suppose we are considering an object lying on an inclined plane.
In which direction will the friction be acting?
Friction will oppose the direction in which motion is likely to take place.
Therefore, in the absence of other forces, it will act up the plane.

As we need to find the normal reaction force to calculate F_{max}, and that force acts perpendicular to the flat surfaces in contact, it is usually best to begin by resolving all forces acting into components parallel and perpendicular to the plane.

Example 2

A crate of mass 30 kg is lying on a plane inclined at an angle of 25° to the horizontal.
The coefficient of friction between the two surfaces is 0.28. (Take $g = 9.8\ \mathrm{m\,s^{-2}}$.)
The crate is in limiting equilibrium, with a force, P, acting together with friction to prevent the crate slipping.
What is the least value of P?



If P has its least value, the friction must take its highest value, F_{max}.

$\perp$ to plane: $R = 30g \cos 25°$

$R = 30 \times 9.8 \times \cos 25°$

$R = 266$ N (3 s.f.)

Since in limiting equilibrium: $F = F_{max} = \mu R$

$= 0.28 \times 266$

$= 74.6$ N (3 s.f.)

// to plane: $P + F_{max} = 30g \sin 25°$

$P = 30 \times 9.8 \times \sin 25° - F_{max}$

$P = 124.2 - 74.6$

$P = \underline{49.6\ \mathrm{N}}$ (3 s.f.)

(As always, although the values have only been written down to 3 s.f., the full accuracy of the calculator was used at each stage in the calculation and the numbers have only been rounded when they have been written down.)

Problems involving friction

Now that we have established the mathematical model for friction, it can crop up in any of the kinds of situations we have already met.

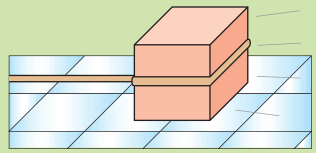
In most cases the best approach is to:
- resolve perpendicular to the plane where friction is occurring to find R,
- resolve parallel to this plane and use $F_{max} = \mu R$.

The following examples include friction with each of Newton's Laws.
Can you remember all three of Newton's Laws?

Back p. 39

Example 3

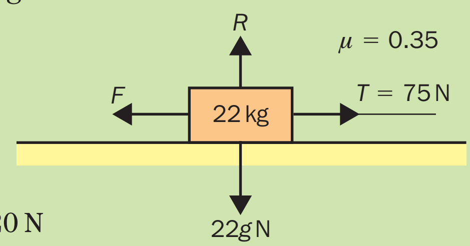
A large box of mass 22 kg is pulled across a horizontal floor by a horizontal rope. The coefficient of friction between the floor and the box is 0.35 and the tension in the rope is 75 N. (Take $g = 10\text{ m s}^{-2}$ and give final answers to 2 s.f.)



a) Will the speed of the box be constant?

b) Suppose the rope is raised to an angle of 30° with the horizontal.
If the tension remains the same, how will the motion of the box change?

a) Newton's First Law says that an object will continue to travel at a constant speed in the same direction if no resultant force is acting.
Is there a resultant force in any direction?



V: since there is no motion vertically: $R = 22g$ N

$R = 22 \times 10 = 220$ N

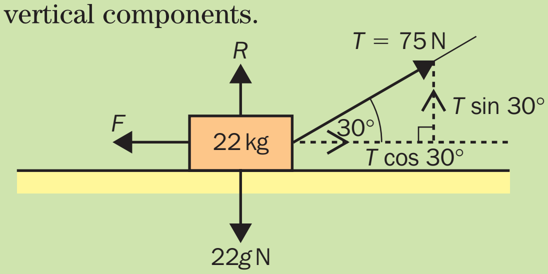
Now, we find the maximum value of friction: $F_{max} = \mu R$

$= 0.35 \times 220$

$= 77$ N

If $T = 75$ N, friction will also be 75 N.
As the forces balance in both directions, the box will be travelling at a constant speed.

b) Since the rope is at an angle, the tension will have horizontal and vertical components.



V: $R + T\sin 30° = 22g$

$R = 22 \times 10 - 75 \times \sin 30°$

$= 220 - 37.5$

$= 182.5$ N

H: friction: $F_{max} = \mu R$

$= 0.35 \times 182.5$

$= 63.875$ N

component of T: $T\cos 30° = 75 \times \cos 30°$

$= 64.952$ N

As the component of tension exceeds the maximum value of friction, we need to use Newton's Second Law to find the acceleration produced, by considering the resultant force in the horizontal direction:

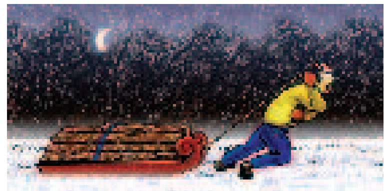
H: Using $F = ma$: $64.952 - 63.875 = 22a$

$1.08 = 22a$

$\therefore \quad a = 0.049$ (2 s.f.)

The box will accelerate in the horizontal direction at a rate of 0.049 m s^{-2}.

Why was there less friction when the angle of the rope was 30° upwards? The upwards pull is reducing the pressure of contact between the rough surfaces. The upwards component is trying to pull the surfaces apart.



Finding the coefficient of friction

What information do we need to work out the coefficient of friction?
We can calculate μ provided we know both R and F_{max}.
What does this tell us about the physical situation?
It must be either in limiting equilibrium or in motion, since $F = F_{max}$.

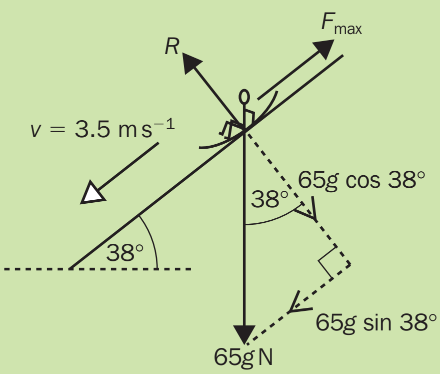
Calculating the value of μ:

Since $\quad F_{max} = \mu R, \qquad$ then $\qquad \mu = \dfrac{F_{max}}{R}$

providing the system is in limiting equilibrium or motion is taking place.

Example 4

A young girl is sliding down a fairground slide on a mat. The mass of the child is 65 kg and the sloping part of the slide makes an angle of 38° with the horizontal direction.
If she is travelling at a constant speed of 3.5 m s^{-1}, what is the coefficient of friction between the slide and the mat?



$\perp$ to plane: $\qquad R = 65g \cos 38°$

// to plane: $\qquad F_{max} = 65g \sin 38°$

Since in motion: $\qquad F = F_{max} = \mu R$

To find μ: $\qquad \mu = \dfrac{F_{max}}{R}$

$$\mu = \frac{65g \sin 38°}{65g \cos 38°}$$

$$\mu = \frac{\sin 38°}{\cos 38°}$$

$$\mu = \tan 38° \qquad \left(\text{since } \frac{\sin\theta}{\cos\theta} = \tan\theta\right)$$

$$\mu = \underline{0.781} \text{ (3 s.f.)}$$

Notice how the value of μ is independent of the mass of the girl. This number would have cancelled from the top and bottom of the fraction whatever it had been.
The value of μ is also independent of the speed, provided there is motion and $F = F_{max}$.

Friction and connected particles

Do you recall the main steps in the method we used before? (See chapter 5.)
We need to write an equation of motion for each particle and add these equations to eliminate the tension.
We do the same here, but resolve perpendicular to the plane of motion first, to find R, and then use $F_{max} = \mu R$.

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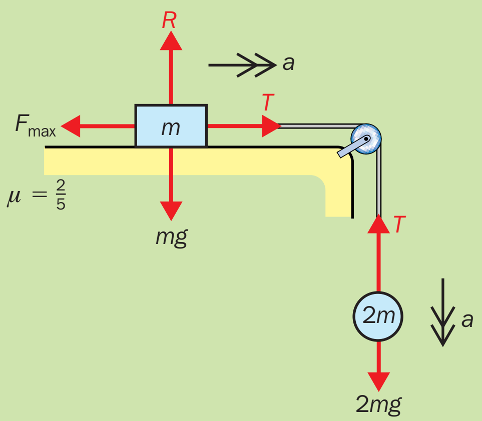
Example 5

A block of mass m kg is resting on a horizontal table. The coefficient of friction between the block and the table is $\frac{2}{5}$. The block is connected by a light, inextensible string via a pulley at the end of the table to a hanging mass of $2m$ kg.

Find, in terms of m, the acceleration of the system when it is released from rest.

Using Newton's Third Law, the magnitude of the tension forces acting on both objects will be the same.



Assuming the system will move, once released: $F = F_{max} = \mu R$.

For the block on the table:

V: $$R = mg$$

Using $F_{max} = \mu R$: $$F_{max} = \tfrac{2}{5}mg$$

H: $$T - F_{max} = ma$$

$$\therefore \quad T - \tfrac{2}{5}mg = ma \qquad \text{..............................(1)}$$

For the hanging mass:

V: $$2mg - T = 2ma \qquad \text{..............................(2)}$$

(1) + (2): $$2mg - T + T - \tfrac{2}{5}mg = ma + 2ma$$

$$2mg - \tfrac{2}{5}mg = 3ma$$

$$\tfrac{10}{5}g - \tfrac{2}{5}g = 3a$$

$$\tfrac{8}{5}g = 3a$$

$$\tfrac{8}{15}g = a$$

Therefore, on release from rest, the acceleration of the system will be $\frac{8}{15}g\ \mathrm{m\,s^{-2}}$

Summary

- **Friction** is a **resistance force** which acts between two surfaces in contact.
- Friction always **opposes motion** (i.e. acts in the opposite direction and prevents motion if possible).
- The maximum value of friction is proportional to the **Normal Reaction Force**.
- Friction only takes its maximum value, F_{max}, when the system is in limiting equilibrium or motion is taking place.
- Friction only needs to be just great enough to prevent motion if a small force is applied.
- Friction can take the following range of values:

$$0 \leqslant F \leqslant F_{\text{max}}, \qquad \text{where} \qquad F_{\text{max}} = \mu R \qquad (R \text{ is the normal reaction force})$$

- Since $\quad F_{\text{max}} = \mu R, \quad$ then $\quad \mu = \dfrac{F_{\text{max}}}{R}$

End of Chapter Questions

1 Can you complete these sentences?

a) Friction acts between … in contact.

b) Friction will only be great enough to … motion.

c) An object just on the point of moving is said to be in … equilibrium.

d) The description … means no friction is acting.

e) Friction is proportional to the … force.

f) μ is the … of friction.

g) Friction cannot cause … to take place.

h) The direction of friction is always opposite to … .

2 Suppose a body is sliding at a constant speed down an inclined plane. Name two important quantities to which the coefficient of friction is independent.

3 What is the minimum value of the frictional force?

4 If $R = 60$ N and $\mu = 0.24$, what is the value of F_{max}?

5 If $F_{\text{max}} = 43$ N and $R = 120$ N, what is μ?

6 If $F_{\text{max}} = 111.6$ N and $\mu = 0.372$, what is R?

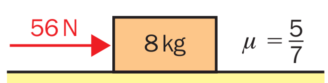
7 A tin of paint with mass 6 kg is placed on a wooden workshop bench. The coefficient of friction between the tin and the bench is 0.4. If a horizontal force of 20 N is applied to the tin of paint, will it move? (Take $g = 9.8 \text{ m s}^{-2}$.)

8 A wooden log of mass 95 kg is being hauled up a slope inclined at 12° to the horizontal. The coefficient of friction between the log and the slope is 0.53. The rope attached to the log is parallel to the slope. What value must the tension in the rope take for motion with constant speed to take place? (Take $g = 9.8 \text{ m s}^{-2}$)

9 A block is at rest on a rough horizontal plane and the coefficient of friction between the block and the plane is μ. The block is subjected to a horizontal force. Find, in each case,

a) the value of the frictional force,

b) the acceleration of the block.

i)


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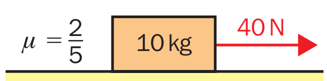

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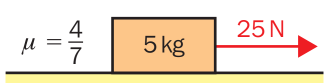

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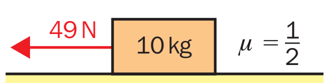
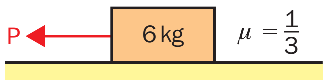
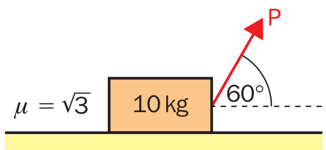


10 In each case, the block shown is in limiting equilibrium, on a rough horizontal plane and the coefficient of friction between the block and the plane is μ. Find P in each case.

a)



b)



11 A box of mass 2 kg is placed on a rough plane which is inclined to the horizontal at an angle θ. The coefficient of friction between the box and the plane is 0.5. Find the maximum value θ for which the box will remain at rest.

12 A particle of mass 4 kg is held in equilibrium on a rough plane which is inclined to the horizontal at an angle of 20°, by a horizontal force of magnitude 5 N. Given that the particle is on the point of slipping down the plane, find, to 3 s.f. the coefficient of friction between the particle and the plane.

13 A trunk of mass 50 kg is at rest on a rough horizontal plane. The coefficient of friction between the plane and the trunk is 0.4. The trunk is then dragged at a constant speed across the plane by a rope which is attached to the trunk and makes a constant angle of $\tan^{-1}\left(\frac{3}{4}\right)$ with the horizontal. Find

a) the normal reaction between the trunk and the plane,

b) the tension in the rope.

14 A particle of weight 30 N is held in equilibrium on a rough plane, which is inclined to the horizontal at an angle of 45°, by a force of magnitude P acting parallel to and up the plane. Given that the coefficient of friction between the particle and the plane is $\frac{1}{3}$, find the complete range of possible values of P.

How to make the Examiner happy

- In general, and on your diagram, use F for friction. Only write F_{max} once you have stated or shown that the system is in limiting equilibrium.
- Where $F \leqslant \mu R$, state why and give its magnitude (equal to the opposing force).
- Use the terms ‘smooth’ and ‘rough’ in your solution as appropriate.

THIS CHAPTER IS ONE PAGE SHORT FOR DOUBLE PAGE SPREADS.
PLEASE ADVISE.

8 Moments

If you have never studied mechanics before, you probably will not guess what a *moment* is.

This is a case where an everyday word is used in mechanics to mean a very specific mathematical idea.

Try the following experiment.
Close this book and place it flat on the table in front of you.
Push it with your finger in the middle of one side.
Can you make it move in a straight line without turning at all?
It may be difficult, but it's not impossible.
Now try to do the same, but with your finger pushing at one of the corners.
What happens now?
It's always going to turn, isn't it?!

The topic of moments is all about the way a force can have a turning effect, depending on where it is acting.

In this chapter you will learn:
- about forces acting on rigid bodies,
- how to calculate the moment of a force,
- what happens around pivots,
- how to describe the sense of a moment,
- how to make moments balance.

The moment of a force

What do we mean by a 'particle model'?
This is when the object in question is considered to be reduced to a single point.
One of the real-life features that we ignore in this case is rotation.
A single force acting on a particle will produce motion in the same direction as the force itself.

Of course, in the real world, the objects that we deal with are larger than single points and their shapes can make a difference to the way they behave if a force is applied.

What model do we need for different shapes?
We use the 'rigid body' model, in which the lengths and distances are used but considered to be fixed and unchanging.

In the experiment at the start of this chapter, the place where the force is applied to the book makes a real difference to the outcome.
The force applied at the corner produces a turning effect. However, the force acting in the middle of a side is able to move the book in a straight line.

The moment of a force is the name we give to the ability of the force to produce **a turning effect** on the rigid body on which it is acting.
(In physics, the word **torque** is used to describe the same concept.)

Two specific mechanical quantities affect the size of the moment of a force. What do you think they are?

The *size of the force* is one of the factors. Clearly, the larger the magnitude of the force, the greater its potential to turn a rigid body.
The other factor is the *distance at which the force is acting*, measured from the point we are considering as the centre of rotation for the turning effect.

Imagine you are walking along a plank of wood, which is resting against a log. Once you have passed over the point where it is balanced, you know it will soon tip. The further you walk along the plank, the sooner you expect it to tip!

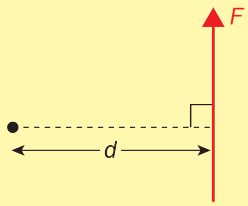
Providing we are using the usual SI units, the moment of a force is simply the magnitude of the force multiplied by the distance at which the force is acting *from* the possible centre of the rotation.
This point may also be called the **pivot** or **fulcrum**, or it could be a **hinge**.

The moment of a force about a point:

$$M = Fd$$

Given that:

M = the moment of the force (N m)
F = the magnitude of the force (N)
d = the perpendicular distance of the line of action of the force from the point (m)

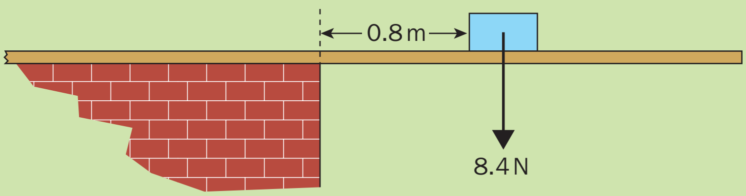


Notice that the distance is measured perpendicular to the line of action of the force.
We will look at a simple case first. The method for dealing with a force acting at an angle other than 90° will be explained later.

Example 1

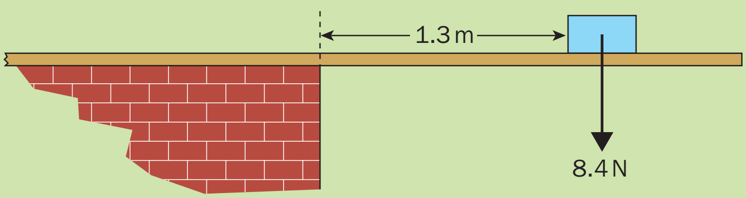
A wooden plank overhangs the end of a brick wall. A lunchbox with a weight of 8.4 N is put on it. Find the moment about the end of the wall if the lunchbox is a) 0.8 m and b) 1.3 m from the wall.

a) Here, the force is the weight of the lunchbox.



Moment about the end of the wall: $M = 8.4 \times 0.8 = \underline{6.72\text{ N m}}$

b) Now, with the lunchbox in the other position.



Moment about the end of the wall: $M = 8.4 \times 1.3 = \underline{10.92\text{ N m}}$

Taking moments

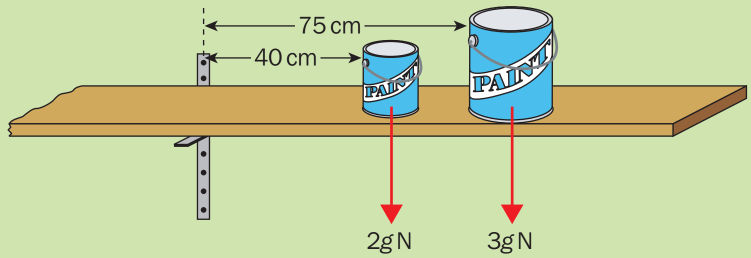
When we want to find the overall turning effect of one or more forces acting on a rigid body, the process we use is called *taking moments*.

If there are a number of forces acting, we simply add together the separate moments to obtain the overall moment.
If there are more forces acting, will that always mean that the total moment is bigger? Can you think of any cases where it might not be true?
The following examples will take you through the different kinds of things that can happen, including some special cases.

Example 2

Two cans of paint, with masses 2 kg and 3 kg, are placed on the end of a shelf, at distances 40 cm and 75 cm, respectively, from the supporting bracket. (Take $g = 9.8\ \mathrm{m\,s^{-2}}$.)

Find the total moment produced by the weights of these two cans.



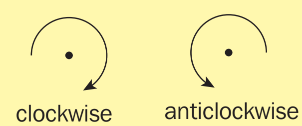
Take moments about $\overset{\frown}{\mathrm{A}}$: ($\overset{\frown}{\mathrm{A}}$ means 'taking clockwise moments about point A')

$$\begin{aligned} \text{Total moment,} \quad M &= 2g \times 0.4 + 3g \times 0.75 \quad \text{(NB: the lengths have been converted to metres.)} \\ &= 2 \times 9.8 \times 0.4 + 3 \times 9.8 \times 0.75 \\ &= 7.84 + 22.05 \\ &= \underline{29.89\ \mathrm{N\,m}} \end{aligned}$$

What terms do we use to describe the two different directions that a shape in a flat plane can rotate?
You probably remember from studying transformations that a rotation can be either **clockwise** or **anticlockwise**.
It is reasonable to use the same language when we refer to moments.

The sense or direction of a moment:

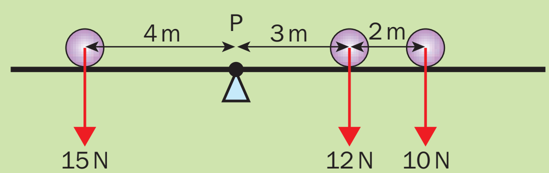
A force may produce a moment that is either **clockwise** or **anticlockwise**.



If clockwise is taken as the positive direction, then:
- a clockwise moment is positive
- an anticlockwise moment is negative.

Example 3

What is the total clockwise moment, about P, of the three forces acting at the distances shown in the diagram?



Let's work out the moment about P for each force in turn.
As we are finding the clockwise moment, any anticlockwise moment will be counted as negative.

$$\begin{aligned} \overset{\frown}{\mathrm{P}}: \quad M &= -(15 \times 4) + 12 \times 3 + 10 \times 5 \quad \text{(The moment of the first force is anticlockwise.)} \\ &= -60 + 36 + 50 \\ &= \underline{26\ \mathrm{N\,m}} \quad \text{(clockwise)} \end{aligned}$$

Forces at an angle

If the force is not acting in a direction perpendicular to the rigid body, will the turning effect be bigger or smaller?
In fact the greatest turning effect is achieved when the angle is 90°.
If the force is at any other angle, the moment is reduced.

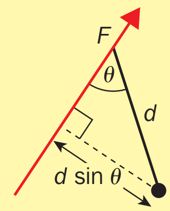
Examination Requirements
Although this method is included for completeness of the topic, the method for finding the moment of a force at an angle will not be tested in the M1 examination.

The moment of a force at an angle:

$M = Fd \sin \theta$

where θ is the angle between the rigid body and the force.

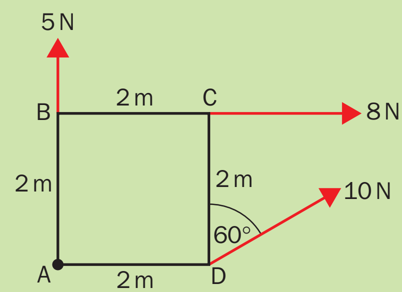
The perpendicular distance from the point to the force is $d \sin \theta$.



Example 4

A square lamina of side length 2 m is hinged to a smooth flat surface at one corner, A.
It is free to rotate smoothly about this point.
Find the total clockwise moment, about A, of the three forces acting as shown in the diagram.

(HINT: It helps to extend the line of action of the angled force and draw in a perpendicular line from the pivot point.)



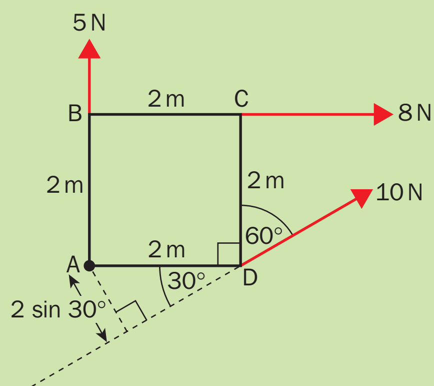
Next, take moments about A, considering each force in turn.

$\curvearrowright$A: $\quad 5 \times 0 + 8 \times 2 + -(10 \times 2 \sin 30°)$

$= 0 + 16 - 20 \times 0.5$

$= 16 - 10$

$= 6$ N m (clockwise)



Did you notice that one of the moments in the last example was zero?
Why was that?
It was because the line of action of the force went straight through the point about which moments were being taken.
In mathematical terms, the moment was zero because the distance from the point to the force was also zero.
A commonsense explanation would be that a force pulling directly on the pivot can't make the lamina rotate either one way or the other.

A force through a point:

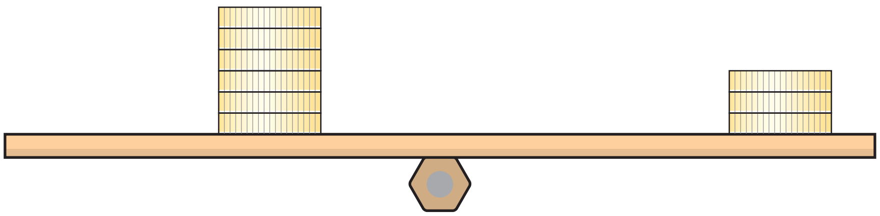
The moment of a force about a point through which it is acting is zero.

Since $\quad d = 0, \quad$ then $\quad M = F \times 0 = 0$

Balancing

How can we seat an adult and a child on a seesaw so that it will balance in the horizontal position? Have you ever tried this?
The answer is that the adult needs to be closer to the pivot than the child.

Try the following practical experiment. Balance a 30 cm ruler on a pen or pencil with flat sides. Take 9 identical coins and place a pile of 3 on one side and a pile of 6 on the other side. Move the piles about until the ruler balances. What do you notice?



TAKE IN A/W A8.5
see-saw
TO BE SUPPLIED

The small pile should be twice as far away from the centre as the large pile. We know that the moment of each pile of coins will be the product of its weight and the distance from the pivot. For these to be equal, the pile that is twice as big will have to be half the distance compared to the small pile.

TAKE IN A/W A8.7
see-saw (balanced)
TO BE SUPPLIED

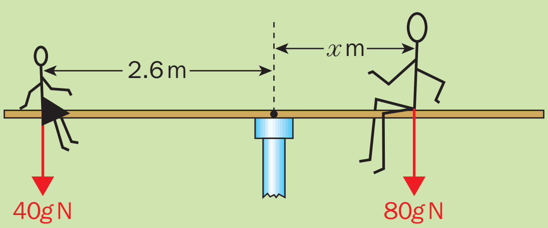
Now let's try to apply that to the two people on a seesaw.

In example 5, one person will cause a clockwise moment about the central pivot and the other an anticlockwise moment. In general, the moments in the two different directions will have to be equal for balancing.

Example 5

A man of mass 80 kg and his daughter of mass 40 kg are trying to balance on a seesaw.
If the girl is 2.6 m from the fulcrum, find the distance of her father from the same point.

Let the man's distance be x.
Remember that we need to consider forces to calculate moments, so we must use their weights and not just their masses. (Take $g = 10\,\text{m s}^{-2}$.)



For the girl: moment $= 40g \times 2.6$

$= 1040\,\text{N m}$ (anticlockwise)

For the man: moment $= 80g \times x$

$= 800x\,\text{N m}$ (clockwise)

For the two to balance:

$$800x = 1040$$
$$x = 1040 \div 800$$
$$x = \underline{1.3\,\text{m}}$$

The man will be 1.3 m from the fulcrum. This is exactly half as far as his daughter.

What will happen if we simply calculate the total moment in (say) the clockwise direction?
Since any anticlockwise moment will count as negative, then the total will be zero, if the moments in the two directions are equal in magnitude.

For the overall turning effects to balance:

Either total clockwise moment = total anticlockwise moment

or total moment about any point must be zero

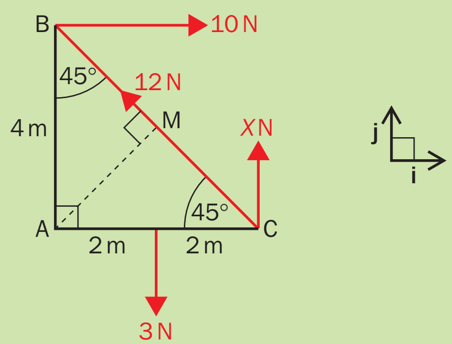
Parallel Forces
Only questions featuring parallel forces will be included in the examination. Example 6 is included for completeness and interest.

In the following example the first approach has been used, as it is easy to see which forces will produce clockwise moments and which will produce anticlockwise moments.

Example 6

Four forces as shown act upon a lamina in the shape of an isosceles right-angled triangle, whose perpendicular sides are 4 m in length.
The moments of the forces balance each other out.

By equating the clockwise and the anticlockwise moments, find the magnitude of the force labelled X.
Find also the resultant force acting on the lamina.

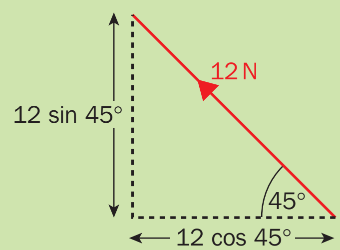


(NB: AM = 4 sin 45°)

Equating moments about A:

$$\begin{aligned} \text{sum of clockwise moments} &= \text{sum of anticlockwise moments} \\ 10 \times 4 + 3 \times 2 &= 12 \times 4 \sin 45° + X \times 4 \\ 40 + 6 &= 48 \sin 45° + 4X \\ 46 - 48 \sin 45° &= 4X \\ (46 - 48 \sin 45°) \div 4 &= X \\ X &= \underline{3.01\text{ N}}\ (3\text{ s.f.}) \end{aligned}$$

We can find the resultant force in vector form, taking the **i** and **j** unit vectors as shown in the diagram above.
The 12 N force needs to be resolved into components.



$$\begin{aligned} \text{Resultant force} &= 10\mathbf{i} + 3.01\mathbf{j} - 3\mathbf{j} + (-12 \cos 45°\,\mathbf{i} + 12 \sin 45°\,\mathbf{j}) \\ &= (10 - 12 \cos 45°)\mathbf{i} + (3.01 + 12 \sin 45° - 3)\,\mathbf{j} \\ &= \underline{(1.51\mathbf{i} + 8.50\mathbf{j})\text{ N}}\ (3\text{ s.f.}) \end{aligned}$$

From these two results we can tell that the lamina is not rotating as the total moment is zero, but that it will be moving and accelerating in a direction parallel to the resultant force.

Maths in Action: How High Can You Stack?

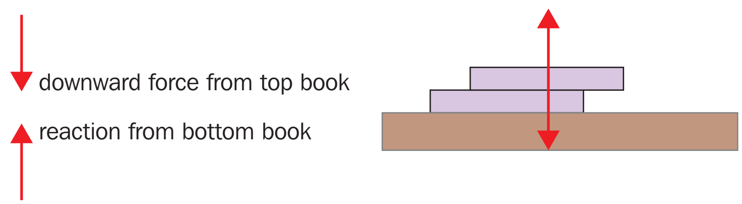
Librarians worry about it. So do school teachers and the people who run bookshops. It's the problem of how high you can stack a pile of books before they topple over.

PICTURE TO BE INSERTED
TO BE SUPPLIED

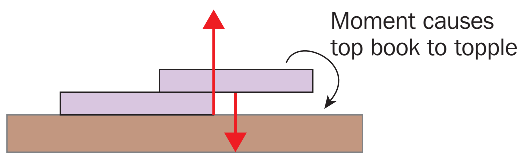
If you place a single book on a desk, it will of course be stable. If you then place a second book on top, that too might be stable, unless it projects too far over the edge of the book below.

Where is the point at which the top book will topple over the edge? Your intuition might tell you that the tipping point is when more than half of the top book is projecting over the bottom book. This can be explained using some simple mechanics.

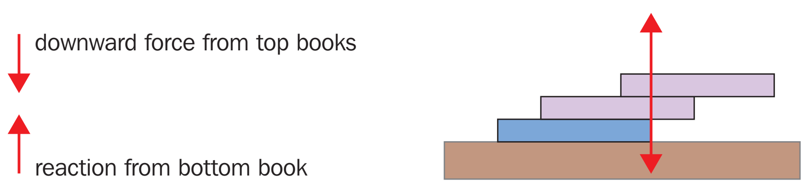
The downward force on the top book is distributed evenly across its area, but fortunately this force can be represented at a single point, known as its centre of mass, which makes the analysis simpler. If the mass is evenly distributed, this will be halfway down the length of the book. Newton's third law says that there will be an equal and opposite reaction from the book below pushing upwards, which can also be represented as acting through a single point. In the first diagram, these forces are in equilibrium.



When the top book projects more than half way over the bottom one, the upward force from the bottom book can no longer line up with the downward force on the top book (it can't push up beyond its edge!). The unaligned forces create a moment (or turning effect), causing the top book to topple over.



What happens if we add a third book? We already know that the top book won't topple if it projects less than halfway over the book below it. If we then imagine the top two books being glued together, the centre of gravity of the two books will be $\frac{1}{4}$ of the way along the top book. Using the same principle as before, the stack of books will be stable so long as the centre of gravity of the top two books does not project beyond the edge of the bottom book.



This means that the top book of the three can project $\frac{3}{4}$ of its width over the bottom book without the pile toppling over.

The calculations for four or more books are rather more complicated, but the principle of calculating the combined centre of mass of the books still applies.

It turns out that there is a surprising pattern in the amount by which the top book can project beyond the bottom one. There is a series known as the harmonic numbers which is defined as follows:

$$H_N = 1 + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} + \ldots + \frac{1}{N}$$

The proportion by which the top book in a pile of N books can project over the bottom book turns out to be be $\dfrac{H_{(N-1)}}{2}$.

So, if there are two books, the top one can overhang by $\tfrac{1}{2}$

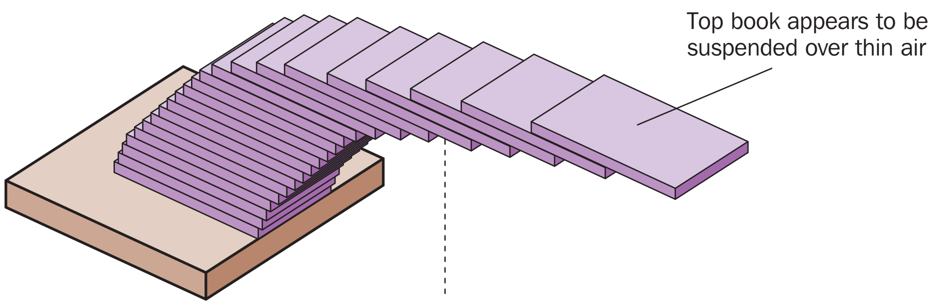
In a pile of three books, the top one can overhang by $\dfrac{(1 + \tfrac{1}{2})}{2} = \tfrac{3}{4}$

In a pile of four books, the top one can overhang by $\dfrac{(1 + \tfrac{1}{2} + \tfrac{1}{3})}{2} = \tfrac{11}{12}$

And in a pile of five books, the top one can overhang by $\tfrac{25}{24}$, which is more than 1!

In other words it projects *beyond* the bottom book.

In fact, if you build a stack that is high enough, it is theoretically possible for the top book to project out as far as you want, making a smoothly curving tower. Architects or sculptors could use this principle to build a leaning tower that didn't require any joints … though a gust of wind or a flock of seagulls landing on the top layer could bring the whole thing tumbling down!



Summary

- The **moment** of a force is the ability of the force to produce a turning effect on a rigid body.
- The formula for working out the moment of a force is: $M = Fd$, where d is the perpendicular distance from the point to the line of action of the force.
- **Taking moments** is the process of finding the overall turning effect of one or more forces acting on a rigid body.
- A force may produce a moment that is either **clockwise** or **anticlockwise**.
- For a force at an angle of θ to the rigid body, $M = Fd \sin \theta$. (Not required for M1 examination)
- The moment of a force about a point through which it is acting is zero.
- The turning effects of the forces acting will balance out if
  either total moment about any point is zero
  or total clockwise moment = total anticlockwise moment.

End of Chapter Questions

1 Can you complete these sentences?

a) The moment of a force is its ability to … a rigid body.

b) The moment is the product of the size of the force and …

c) The two possible directions of a moment are … and … .

d) The moment of a force about a point through which it acts is …

e) Finding and adding all the moments about a certain point is called …

f) For forces at an angle, we use … to find the perpendicular distance.

g) For moments to balance, the total clockwise moment must equal …

2 The pivot or hinge is also called the …

3 What is the moment of a force of 3.8 N, acting at a perpendicular distance of 1.5 m from the pivot?

4 A rod AB of length 6 m is hinged at its centre, C. There is a mass of 24 kg at A. What mass must be placed at a distance 2 m from the other end, B, in order to balance this?

5 A monkey of mass 3.9 kg moves along a plank of wood which rests on and extends from a wall. If the moment about the end of the wall exceeds 75 N m, the plank will tip. How far out can the monkey safely go?

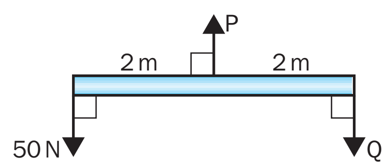
6 The following forces are acting perpendicular to a light rod PQ of length 4 m; 8 N upwards at a distance of 0.6 m from P, 5.2 N downwards at 3 m from P, 1.9 N upwards at 3.8 m from P. Find the total moment about P.

7 A tyre of mass 28 kg is placed on one end of a seesaw 8 m long. Where must a boy of mass 56 kg sit in order to balance the seesaw horizontally?

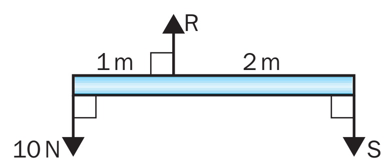
Where necessary, in questions 8–12, assume that the weight force acts downwards from the centre of a rod, plank or beam.

8 In each of the following diagrams a light rod is in equilibrium under the action of forces acting perpendicular to its length. Find, in each case, the magnitude of the unknown forces.

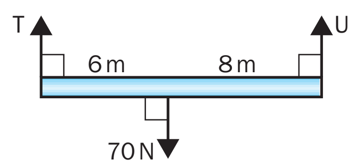
a)



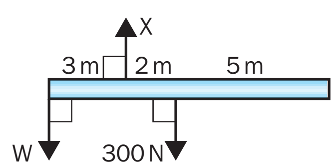
b)



c)



d)



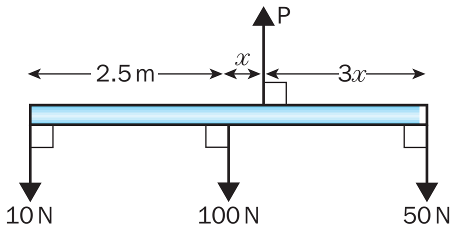
9 A uniform rod AB of length 8 m and mass 50 kg rests on a support at the point C of the rod where $AC = 3$ m. A particle of mass m is attached to the rod at the point A and as a result the rod can rest in equilibrium in a horizontal position.

a) Find the value of m.

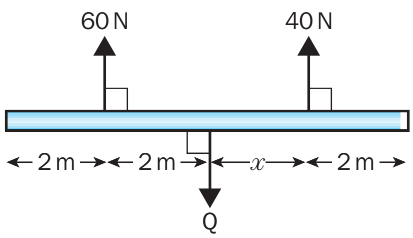
b) Find the magnitude of the force acting on the rod at the point C.

10 In each of the following diagrams a light rod is in equilibrium under the action of forces acting perpendicular to its length. Find, in each case, the value of x.

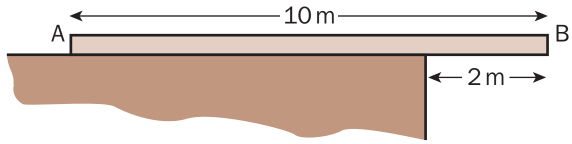
a)



b)



11 A uniform plank AB of length 10 m and weight 560 N rests on a horizontal raised platform. The end B overhangs the edge of the platform by 2 m. A man of weight 1400 N walks from A to B.



a) Find the distance of the man from B when the plank is about to tilt.

b) Find the smallest weight W newtons which can be placed on the plank at A in order to enable the man to walk safely all the way along the plank without it tilting.

12 A uniform gymnast's beam PQ of length 6 m and weight 180 N is supported in a horizontal position by two vertical ropes. The ropes are attached to the beam at points R and S, where $PR = 1$ m and $SQ = 2$ m.

a) Find the tension in each of the two ropes.

When a weight W is placed on the beam at the end Q, the beam is on the point of tilting.

b) State the value of the tension in the rope attached at R.

Find

c) the tension in the rope attached at S,

d) the value of W.

How to make the Examiner happy

- We always write the formula for the moment as $M = F \times d$. In your calculations, always put the magnitude of the force first and the distance afterwards.
- Make it really clear if you are taking clockwise or anticlockwise moments.
- Indicate the point about which you are taking moments on your diagram.
- State the direction of the moment with your final answer.

9 Statics

What do we mean when we say something is *static*?
We would normally use this word to mean stationary or still, as opposed to moving. It could also mean 'unchanging' in some other respect.
The *Financial Times* might say that 'share prices remain static' if they have kept the same value.

In mechanics, the study of statics refers to the conditions that keep a particle or rigid body from moving. It will involve forces of all kinds, such as weight, reaction, tension, thrust and friction. We will also be calculating moments of forces in some cases. We have met all of these ideas before.

This final chapter therefore gives us the opportunity to bring together ideas from many of the areas of mechanics that we have studied so far.
You might like to look back at the chapter summaries from earlier in the book before continuing with this topic.

In this chapter you will learn:
- the meaning of equilibrium,
- how to find and use the resultant force,
- how to resolve and equate forces in perpendicular directions,
- how to calculate and use the total moment,
- the general conditions for the equilibrium of a rigid body.

Equilibrium of a particle

Equilibrium is a word which normally refers to a sense of balance or rest.
What could equilibrium mean in the context of mechanics?
There are two types of equilibrium in mechanics.

The obvious kind, which involves an object remaining stationary at rest under the action of a system of forces, is called **static equilibrium**.
This is the sort of equilibrium dealt with in this chapter.

Back p. 39

There is another type of equilibrium, in which the forces balance, but the body in question is not at rest. Do you remember Newton's First Law?
It says: 'A body will continue to remain at rest or *move at constant speed in a straight line* unless an external force makes it act otherwise'.
This is called **dynamic equilibrium** and, as the forces are in balance, there is no acceleration. However, the body continues to move with constant speed in a straight line.
In both kinds of equilibrium, there are either ***no external forces acting*** or the ***forces balance out***.

If all the forces acting are in the same two-dimensional plane, then they can be described as a system of **coplanar** forces. (We could use **i** and **j**.)
The problems we will be looking at will all be either one-dimensional or two-dimensional in nature, although the same theory applies just as well to equilibrium in three dimensions. (We could use **i**, **j** and **k**.)

In the following examples, think what forces are acting and look for ones which must balance out for equilibrium to be preserved.

Example 1

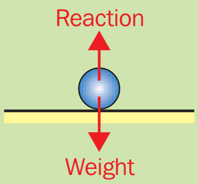
For each of the situations described below, draw a force diagram and state which forces must be equal for the object to remain at rest in equilibrium.

a) A glass paperweight is placed on a horizontal table.

b) Two equally strong tug-of-war teams competing in the gym have reached a standstill.

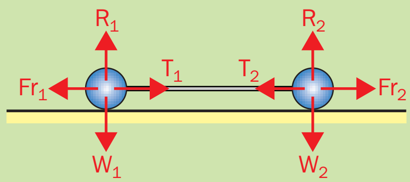
c) A car is parked on a flat road and a child is trying to push it, but it doesn't move at all.

a)



Weight = Reaction

b)

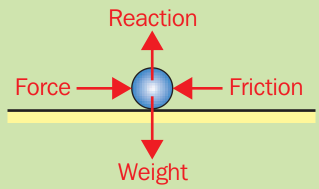


$T_1 = T_2$

$W_1 = R_1$, $W_2 = R_2$

$T_1 = Fr_1$, $T_2 = Fr_2$

c)



Weight = Reaction

Force = Friction

These situations have all involved two or more forces.
Can a particle remain in equilibrium under the action of ***one force***?
No, this is impossible. Newton's Laws tell us that the particle would experience an acceleration in accordance with the rule: $\mathbf{F} = m\mathbf{a}$.
Perhaps we could argue for the rather trivial case where the magnitude of the force is zero! That is really the same as there being no force acting.

Can you describe how it would be possible for a particle to remain at rest in equilibrium under the action of ***two forces***?
The two forces must be equal in magnitude and opposite in direction.
They will also act along the same line, as they will both have to act on the particle itself.

For equilibrium under two forces:
The two forces must be
- equal in magnitude
- opposite in direction
- acting along the same line.

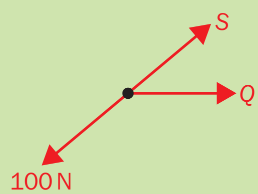
Example 2

a) State the value of each of the lettered forces, if the particle is at rest in equilibrium:

i)

ii)

iii)

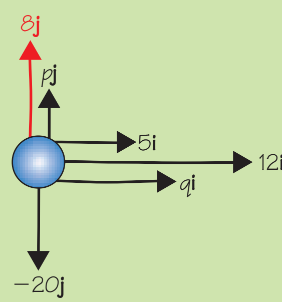


Answers: $R = 50\,\text{N}$ $\quad T = 125\,\text{N}$ $\quad S = 100\,\text{N}$, $Q = 0\,\text{N}$

b) A particle is at rest in equilibrium under the action of the following six forces (all in newtons):

$5\mathbf{i}$, $\quad p\mathbf{j}$, $\quad -20\mathbf{j}$, $\quad 12\mathbf{i}$, $\quad q\mathbf{i}$, $\quad$ and $\quad 8\mathbf{j}$.

Sketch the situation and calculate the values of p and q.



Consider the forces in the **i** direction and the **j** direction separately.

For the forces to balance each other out:

$$q = -(12 + 5) \qquad \text{and} \qquad p = -(-20 + 8)$$
$$q = -17\,\text{N} \qquad\qquad\qquad p = 12\,\text{N}$$

Equating forces

The application of a force will cause a particle to accelerate in proportion to its mass. (This is a form of Newton's Second Law.)
How do we normally state this?
The equation is: $\mathbf{F} = m\mathbf{a}$.

In this equation, **F** is the resultant of any forces acting on a particle. Since the mass of a particle is usually not zero, the only way that the acceleration will be zero is if the force is zero.
This gives us the crucial mathematical condition for equilibrium.

TAKE IN A/W A9.2
finger pushing snooker ball
TO BE SUPPLIED

Condition for equilibrium of a particle (1):

For equilibrium, the resultant force must be zero

i.e. $\mathbf{R} = \mathbf{0}$, where $\mathbf{0}$ is the zero vector.

In unit-vector form: $\mathbf{R} = 0\mathbf{i} + 0\mathbf{j}$

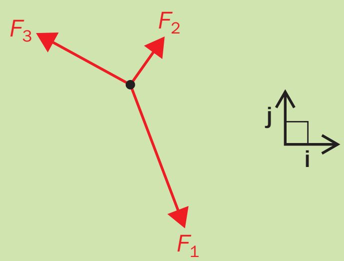
How does this become a useful method?
We find the resultant force acting in a given situation and use the fact that it must be equal to the zero vector for equilibrium to be maintained.

Example 3

A particle is at rest under the action of three forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$.
If $\mathbf{F}_1 = (8\mathbf{i} - 18\mathbf{j})$ N and $\mathbf{F}_2 = (4\mathbf{i} + 5\mathbf{j})$ N, find $\mathbf{F}_3$ in unit vector form and state its magnitude.

It is always a good idea to sketch the problem. (This might help us, for example, to spot an error in the sign of our answer.)

An estimate for $\mathbf{F}_3$ has been drawn in.



In this kind of question, it is easiest to express the unknown vector in component form.
Let $\mathbf{F}_3 = a\mathbf{i} + b\mathbf{j}$

Now apply the condition for equilibrium: $\mathbf{R} = \mathbf{0}$

$$\therefore \quad \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0}$$

$$\therefore \quad (8\mathbf{i} - 18\mathbf{j}) + (4\mathbf{i} + 5\mathbf{j}) + a\mathbf{i} + b\mathbf{j} = \mathbf{0}$$

Collecting similar components together:

$$(8\mathbf{i} + 4\mathbf{i} + a\mathbf{i}) + (-18\mathbf{j} + 5\mathbf{j} + b\mathbf{j}) = 0\mathbf{i} + 0\mathbf{j}$$

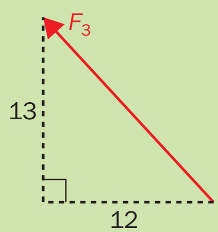
$$(12\mathbf{i} + a\mathbf{i}) + (-13\mathbf{j} + b\mathbf{j}) = 0\mathbf{i} + 0\mathbf{j}$$

$$\therefore \quad a = -12,\ b = 13$$

So: $\mathbf{F}_3 = \underline{(-12\mathbf{i} + 13\mathbf{j})\text{ N}}$

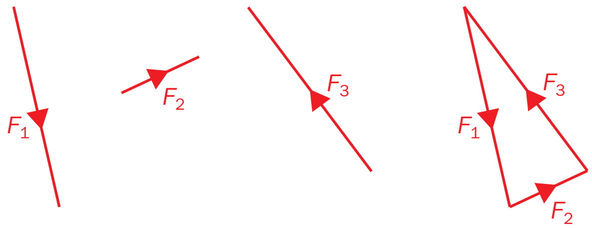
Using Pythagoras' Theorem to find the magnitude:

$$|\mathbf{F}_3| = \sqrt{((-12)^2 + 13^2)}$$
$$= \sqrt{(144 + 169)}$$
$$= \sqrt{313}$$
$$= \underline{17.7\text{ N}}\ (3\text{ s.f.})$$



In example 3, what we did in effect was to choose our values for a and b so that the corresponding components of the resultant force became zero. In fact, this leads to an alternative approach to solving equilibrium problems that we will look at in the next section.

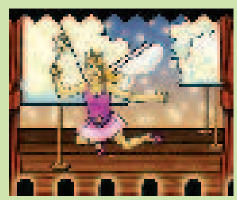
Let's take another look at the solution to the problem in the last example. What will happen if we draw all three forces consecutively on the same diagram?
Since $\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} = \mathbf{0}$, they make a **triangle of forces**.
Why do they make a triangle?
This is because if the sum of three vectors is zero, then when they are placed in sequence, the total 'journey' takes you back to where you start.



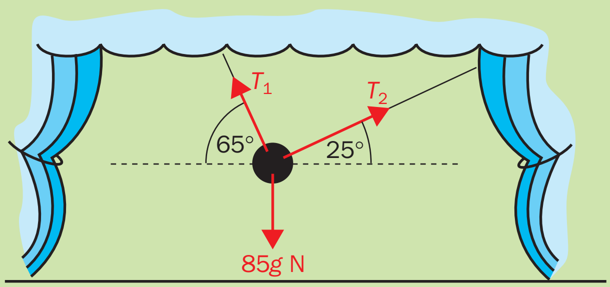
For equilibrium problems involving only three forces we can use this property to obtain our solution.

Example 4

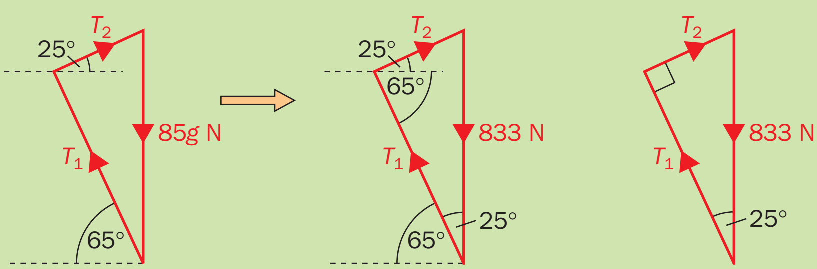
A stage fairy is being flown on wires connected to a harness. At the moment when she is in equilibrium over the middle of the stage, one wire is at an angle of 65° above the horizontal and the other is at 25° above the horizontal. The mass of the fairy is 85 kg. (Take the value of g to be $9.8\ \text{m s}^{-2}$.) Find the magnitude of the tensions in the two wires.



First of all we sketch the situation.



Next we redraw the forces into a triangle, transferring the angles. By using angle facts (e.g. angles between parallel lines) it is possible to identify the angles inside the triangle.



As this is a right-angled triangle, it is easy to calculate the forces required.

$T_1 = 833 \cos 25° = \underline{755\ \text{N}}$ (3 s.f.)

$T_2 = 833 \sin 25° = \underline{352\ \text{N}}$ (3 s.f.)

This was a particularly good question in which to use the 'triangle of forces' approach, because it was a right-angled triangle. However, we could always have used the sine rule or the cosine rule if we had needed to.

Resolving forces

What do we do if there are more than three forces acting and they are not simply horizontal or vertical?
It will be necessary to choose appropriate perpendicular directions and resolve the forces into components in those two directions. We can then ensure that the component of the resultant force in each direction is zero.

If there are a number of forces in different directions, the simplest method may be to resolve **all** the forces into horizontal and vertical components.

When is this not the best approach?
There is the possibility that a majority of the forces act predominantly in a different pair of perpendicular directions.

What is an example of that?
An object on a rough inclined plane will have at least two forces acting perpendicular to each other but not horizontal and vertical.
For problems of this kind it is best to resolve in directions parallel and perpendicular to the plane.

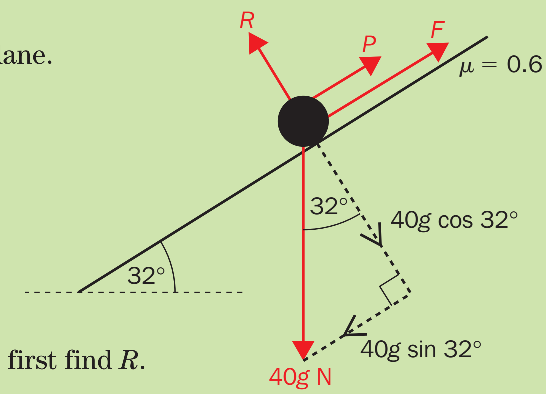
Example 5

An object of mass 40 kg lies at rest on a rough plane inclined at an angle of 32° to the horizontal direction.
The coefficient of friction between the object and the plane is 0.6.
Taking $g = 9.8\ \mathrm{m\,s^{-2}}$, find, correct to 3 significant figures, the magnitude of the least additional force needed to stop the object from sliding down the plane.

As the object is on the point of sliding down the plane, it is in limiting equilibrium with friction acting up the plane.

Let the extra force be P.

The weight is the only force that needs to be resolved into components.



We need to find the value of the frictional force, so let's first find R.

Equating forces $\perp$ to the plane: $R = 40g\cos 32°$

Since in limiting equilibrium:

$$\begin{aligned} F &= F_{\text{max}} \\ &= \mu R \\ &= 0.6 \times 40g\cos 32° \\ &= 199\ \text{N (3 s.f.)} \end{aligned}$$

Equating forces // to the plane:

$$\begin{aligned} P + F_{\text{max}} &= 40g\sin 32° \\ P + 0.6 \times 40g\cos 32° &= 40g\sin 32° \\ P &= 40g\sin 32° - 0.6 \times 40g\cos 32° \\ P &= 8.27\ \text{N (3 s.f.)} \end{aligned}$$

∴ An additional force of 8.27 N acting up the plane will be sufficient to prevent the object from sliding.

What we have just used is a particular application of the condition for equilibrium that the resultant force must be zero.
If the resultant force is expressed in component form, regardless of the direction of the perpendicular components, ***each component will be zero***.

What difference does this make?
Rather than dealing with the resultant as a whole, we can consider the components in each direction separately.

Condition for equilibrium of a particle (2):

For equilibrium, the component of the resultant force in any direction must be zero.

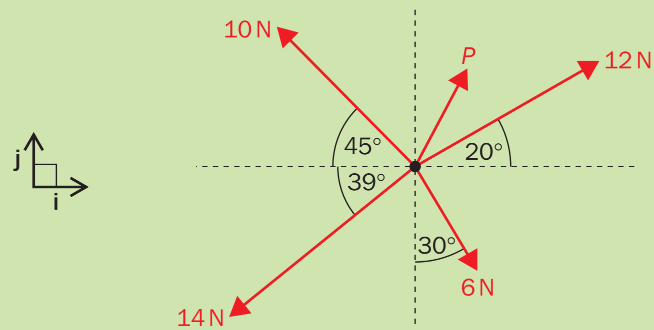
i.e. if: $\mathbf{R} = p\mathbf{i} + q\mathbf{j}$, where the unit vectors act in any suitable perpendicular directions
then: $p = 0$ and $q = 0$

So, we can do either of the following:
- make the components in any direction sum to zero
- equate the positive and negative components in a given direction.

In the following example, as there is no particular pattern to the angles of the forces, we will simply resolve all of them into horizontal and vertical components, using unit vectors.
We can then make sure they sum to zero in each direction.

Example 6

A particle is in equilibrium under the action of the forces shown in the diagram.
What must be the magnitude and direction of **P**?



Let us consider first the horizontal components and then the vertical components, where **i** and **j** are in the directions shown.
Also, let $\mathbf{P} = (a\mathbf{i} + b\mathbf{j})$ N.

i components: $a + 12\cos 20° + 6\sin 30° - 14\cos 39° - 10\cos 45° = 0$

$a = 14\cos 39° + 10\cos 45° - 12\cos 20° - 6\sin 30°$

$a = 3.67$ N (3 s.f)

j components: $b + 12\sin 20° - 6\cos 30° - 14\sin 39° + 10\sin 45° = 0$

$b = 6\cos 30° + 14\sin 39° - 12\sin 20° - 10\sin 45°$

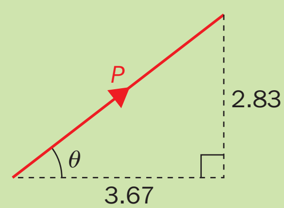
$b = 2.83$ N (3 s.f.)

$\therefore \quad \mathbf{P} = (3.67\mathbf{i} + 2.83\mathbf{j})$ N

By Pythagoras: $|\mathbf{P}| = \sqrt{(3.67^2 + 2.83^2)}$

$= \sqrt{21.5}$

$= \underline{4.64 \text{ N}}$ (3 s.f.)



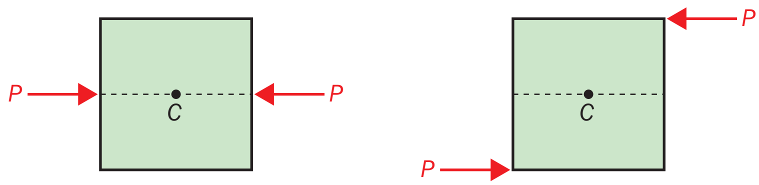
Angle with **i** direction: $\theta = \tan^{-1}(2.83 \div 3.67) = \underline{37.6°}$

$\therefore$ Force **P** has a magnitude of 4.64 N and acts at an angle of 37.6° with the **i** direction.

General equilibrium of a rigid body

Is it possible to have a situation where the resultant force is zero but the object is *not* in equilibrium?

Compare the following two situations.
Equal and opposite forces act on a square lamina in a horizontal plane.
In the first case they act in the middle of opposite sides.
In the second case they act at diagonally opposite corners, as shown.
What will happen?
Try it for yourself with a flat book, pushing with equal force at the two places shown.



In the first instance, the lamina will rest in equilibrium.
However, in the second case, the lamina will rotate about its centre.

What is the mathematical difference?
In both cases the resultant force is zero. But what if we consider moments?
In the first case the moment of both forces about the centre will be zero, but in the second, both forces will have a clockwise moment about the centre.
The result is that the lamina will rotate on the spot.

For equilibrium, we need to be sure that there is **no rotation**, as well as there being no movement through space, i.e. **no translation**.
This means that the total moment of the forces has to be zero as well as the resultant force being zero.

There are therefore two aspects to ensuring equilibrium of a rigid body:

- the forces must balance
- the moments must balance.

Conditions for equilibrium of a rigid body:

the resultant force in any direction must be zero

and

the sum of the moments about any point must be zero

How does this become a practical method?

The two steps of the method correspond to the two different aspects:

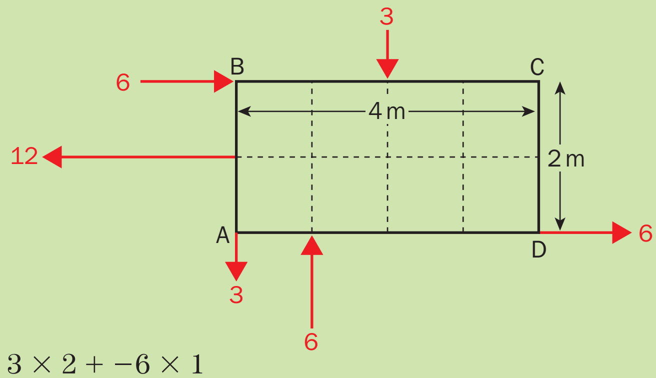
- resolve the forces acting into components in two directions and make sure the component in each direction is zero
- take moments about a suitable point and ensure the sum is zero.

Remember to take moments about the point with the most forces through it, since the moment of each of those forces will be zero.

These final two examples include more forces than might be expected in an examination question, where only parallel forces will be used.

Example 7

A lamina, measuring 2 m by 4 m is acted upon by several forces, as shown in the diagram. (All forces are in newtons.) Show that the lamina will be in equilibrium under the action of these forces.



Sum of horizontal forces: $6 - 12 + 6 = \underline{0}$

Sum of vertical forces: $-3 - 3 + 6 = \underline{0}$

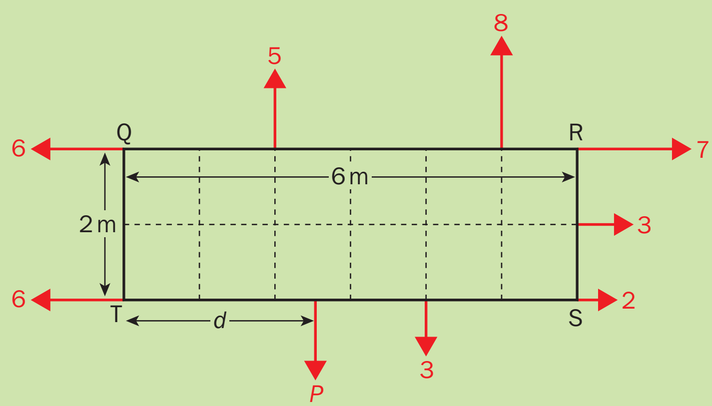
Taking clockwise moments, $\overset{\frown}{A}$:
$$\begin{aligned} M &= -12 \times 1 + 6 \times 2 + 3 \times 2 + -6 \times 1 \\ &= -12 + 12 + 6 - 6 \\ &= \underline{0} \end{aligned}$$

The horizontal and vertical components of the resultant force are both zero, and the sum of the moments is also zero. Therefore the lamina will be in equilibrium.

In the case above, the lamina was already in equilibrium.
Now let's try an example where we must identify the single additional force required to bring about equilibrium.

Example 8

Find the magnitude and position of the additional force, **P**, needed to maintain the equilibrium of the lamina under the action of the forces shown.



As we are trying to achieve equilibrium, we can equate the components of the forces in each direction.

Equating horizontal forces:
$$\begin{aligned} 6 + 6 &= 7 + 3 + 2 \\ 12 &= 12 \end{aligned}$$

The horizontal forces already balance.

Equating vertical forces:
$$\begin{aligned} P + 3 &= 5 + 8 \\ P + 3 &= 13 \\ \therefore \quad P &= \underline{10\,\text{N}} \end{aligned}$$

Taking clockwise moments, $\overset{\frown}{T}$:
$$\begin{aligned} M &= -6 \times 2 + -5 \times 2 + -8 \times 5 + 7 \times 2 + 3 \times 1 + 3 \times 4 + P \times d \\ &= -12 - 10 - 40 + 14 + 3 + 12 + Pd \\ &= Pd - 33 \\ &= 10d - 33 \qquad \text{(since } P = 10) \end{aligned}$$

For the sum of the moments to equal zero:
$$\begin{aligned} 10d - 33 &= 0 \\ 10d &= 33 \\ \therefore \quad d &= \underline{3.3\,\text{m}} \end{aligned}$$

Summary

- **Statics** is the study of stationary particles or rigid bodies.
- **Static equilibrium** is the state of rest where there is no translation or rotation.
- Conditions for **equilibrium of a particle**:
  the resultant force must be zero (i.e. $\mathbf{R} = \mathbf{0} = 0\mathbf{i} + 0\mathbf{j}$)
  or the component of the resultant force in any direction must be zero.
- Additionally, for **equilibrium of a rigid body:**
  the sum of the moments about any point must be zero. (i.e. $M = 0$, about any point)

End of Chapter Questions

1 Can you complete these sentences?

a) The two types of equilibrium are … equilibrium and … equilibrium.

b) The study of how forces keep objects stationary is called …

c) Forces all acting in the same plane are called … forces.

d) The least (non-zero) number of forces required to maintain equilibrium is … .

e) For equilibrium, the … force must be zero.

f) For equilibrium, the … of the resultant force in any direction must be zero.

g) For equilibrium of a rigid body, the sum of the … must be zero.

h) If a body is in equilibrium under the action of three forces, they will form a …

2 If a mass is at rest on a horizontal surface and no other forces are acting, which two forces must be equal in magnitude.

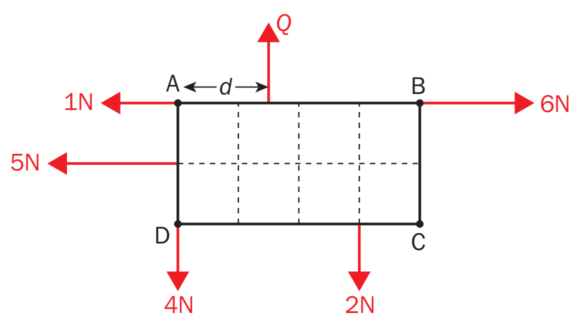
3 Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a rigid body. If $\mathbf{F}_1 = 16\mathbf{i}$ N and $\mathbf{F}_2 = -16\mathbf{i}$ N, what other property is needed to ensure equilibrium?

4 A particle is at rest under the action of three coplanar forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$. If $\mathbf{F}_1 = (21\mathbf{i} - 5\mathbf{j})$ N and $\mathbf{F}_2 = (-8\mathbf{i} + 37\mathbf{j})$ N, find $\mathbf{F}_3$ in unit vector form.

5 Two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a body. A third force, $\mathbf{F}_3$, is added to the system to bring about equilibrium.
If $\mathbf{F}_1 = (-34\mathbf{i} + 25\mathbf{j})$ N and $\mathbf{F}_2 = (22\mathbf{i} - 30\mathbf{j})$ N, find $\mathbf{F}_3$ in unit vector form.
Calculate the magnitude of $\mathbf{F}_3$ and its angle measured clockwise from the $\mathbf{i}$ direction.

6 A paraglider of mass 130 kg (including kit) is descending at a constant speed. If he is supported by two cables, each of which make an angle of 18° outwards from the upward vertical direction, find the magnitude of the tension in each cable. (Take $g = 9.8\ \text{m s}^{-2}$.)

7 A rectangular lamina 2 m by 4 m is at rest in equilibrium under the action of the forces shown in the diagram. Find the magnitude of the force $\mathbf{Q}$ and the distance d of its line of action from A.



8 Each diagram shows a particle of weight 20 N held at rest on a smooth inclined plane by a force $\mathbf{F}$. In each case find the value of $\mathbf{F}$.

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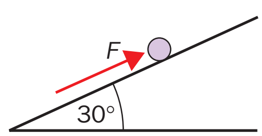

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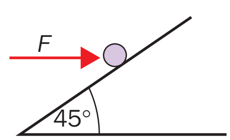


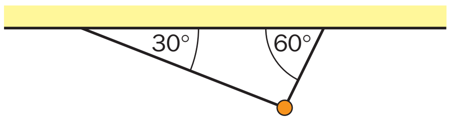
9 exam A body is acted upon by the forces $(4\mathbf{i} - 5\mathbf{j})$ N, $(16\mathbf{i} + 3\mathbf{j})$ N and $(p\mathbf{i} + 8\mathbf{j})$ N. Given that the resultant force is parallel to the vector $(3\mathbf{i} + \mathbf{j})$,

a) find the value of p.

A fourth force **F** is then added so that the body is now in equilibrium,

b) find the magnitude of **F**.

10 exam

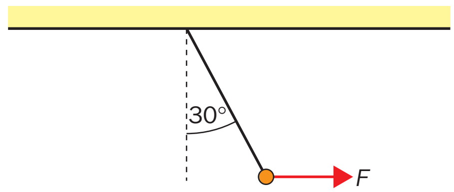


A particle of mass 500 g is attached to one end of each of two light inextensible strings. The particle hangs at rest in equilibrium and the strings make angles of 30° and 60° with the horizontal. Find, to 3 s.f., the tension in each string.

11 exam A particle of weight 40 N is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point on a ceiling. The particle is held in equilibrium, with the string at an angle of 30° with the downward vertical, by a horizontal force **F**.

a) Find, to 3 s.f., the tension in the string.

b) Find, to 3 s.f., the value of **F**.



12 exam A particle is maintained in equilibrium under the action of three forces **P**, **Q** and **R**. The force **P** $= (3\mathbf{i} - \mathbf{j})$ N. Given that **Q** is parallel to the vector $(-2\mathbf{i} + 3\mathbf{j})$ and **R** is parallel to the vector $(\mathbf{i} - 2\mathbf{j})$, find **Q** and **R**.

How to make the Examiner happy

- Make it clear whether you are going to show that the resultant force is equal to zero, or whether the components in each direction total zero.
- Use a letter to represent any unknown lengths, e.g. when a man walks part of the way along a beam.
- As you can choose any point for taking moments, use the one which has the most forces (or any unknown forces) acting through it.

Revision Questions

Bank A

1 Two constant forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are given by $\mathbf{F}_1 = (2\mathbf{i} - 6\mathbf{j})$ N and $\mathbf{F}_2 = (a\mathbf{i} + 2a\mathbf{j})$ N.

a) Find the angle between $\mathbf{F}_1$ and $\mathbf{i}$. **(3)**

The resultant $\mathbf{R}$ of $\mathbf{F}_1$ and $\mathbf{F}_2$ is parallel to $\mathbf{j}$.

b) Find the magnitude of $\mathbf{R}$. **(4)**

2 [*In this question, the horizontal unit vectors* $\mathbf{i}$ *and* $\mathbf{j}$ *are directed due east and due north respectively.*]

At 3 pm cyclist C has position vector $(-9\mathbf{i} + 6\mathbf{j})$ km and is moving with constant velocity $(3\mathbf{i} + 12\mathbf{j})$ km h^{-1} and cyclist D has position vector $(16\mathbf{i} + 6\mathbf{j})$ km and is moving with constant velocity $(-9\mathbf{i} + 3\mathbf{j})$ km h^{-1}.

a) Find how far apart the cyclists are at 3 pm. **(1)**

b) Write down the position vectors of C and D after a further t hours. **(3)**

c) Hence find the vector from C to D after a further t hours. **(2)**

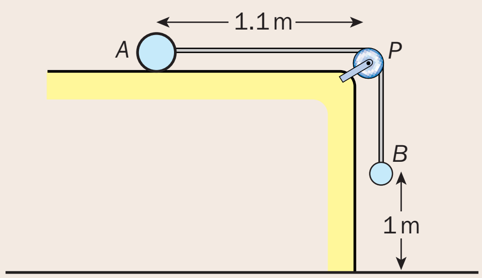
d) At what time will C be due north of D? **(3)**

3 A lorry accelerates along a straight horizontal road from a speed of 14 m s^{-1} to a speed of 34 m s^{-1} in 20 s.

a) Find how far the lorry travels during this 20 s period. **(2)**

b) Find how long it takes to cover half of this distance. **(7)**

4



A particle A of mass $3m$ is at rest on a rough horizontal table. The particle is attached to one end of a light inextensible string which passes over a small smooth fixed pulley P which is at the edge of the table. Another particle B of mass $2m$ is attached to the other end of the string and hangs freely. The line AP is perpendicular to the edge of the table and A, P and B all lie in the same vertical plane. The system is released from rest with the string taut when A is 1.1 m from the edge of the table and B is 1 m from the floor, as shown in the figure. Given that B hits the floor after 2 s and does not rebound

a) find the acceleration of A during the first 2 s of the motion, **(2)**

b) find, to two decimal places, the coefficient of friction between A and the table. **(9)**

c) Determine, by calculation, whether A reaches the pulley. **(7)**

5 A particle P of mass 2 kg is pushed by a constant horizontal force of magnitude 40 N up a line of greatest slope of a rough plane which is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between P and the plane is $\frac{1}{5}$. Find

a) the magnitude of the normal reaction between P and the plane, **(4)**

b) the acceleration of P. **(4)**

6 A uniform rod AE, of length 0.8 m and mass 2 kg, rests horizontally on two smooth supports placed at B and D. Given that $AB = 0.1$ m and $DE = 0.2$ m, find

a) the thrust on the support at B, **(3)**

b) the thrust on the support at D. **(2)**

When a load of mass M kg is attached to the rod at the point A, the rod is about to tilt about the point B. When the load of mass M kg is attached to the rod at a point x cm from E, the rod is about to tilt about the point D. Find the value of

c) M, **(5)**

d) x. **(3)**

7 A particle is suspended by two light inextensible strings and hangs in equilibrium. The first string is inclined at 60° to the horizontal and the tension in that string is 30 N and the second string is inclined at 30° to the horizontal. Find, to 3 significant figures,

a) the weight of the particle, **(3)**

b) the tension in the second string. **(3)**

8 A block of mass 4 kg is placed on a plane inclined at an angle of 30° to the horizontal. The coefficient of friction between the block and the plane is 0.2.

a) Show that the block will slide down the plane. **(4)**

b) Find the magnitude of the *least horizontal* force that is needed to prevent it sliding down the plane. **(7)**

9 A cannon of mass 600 kg lies at rest on a rough horizontal plane. It is used to fire a 2 kg shell horizontally with an initial speed of 300 m s^{-1}.

a) Find the magnitude of the impulse exerted on the shell by the cannon. **(2)**
b) Find the initial speed of recoil of the cannon. **(2)**

Given that the cannon travels a distance of 10.2 cm before coming to rest,
c) find, to 1 decimal place, the coefficient of friction between the cannon and the plane. **(6)**

10 A particle A of mass $2m$ is moving on a smooth horizontal floor with speed u. Another particle B of mass km is moving on the floor with speed $3u$ in the opposite direction. The two particles collide directly and as a result of the collision the directions of motion of both particles is reversed and the speed of A is halved. Find
a) the range of possible values of k, **(5)**
b) the magnitude of the impulse on A from B. **(3)**

Bank B

1 A particle P of mass 0.5 kg is acted upon by two horizontal forces $(2\mathbf{i} + 3\mathbf{j})$ N and $(2\mathbf{i} - 6\mathbf{j})$ N where $\mathbf{i}$ and $\mathbf{j}$ are unit horizontal vectors due east and due north respectively. Find
a) the magnitude of the acceleration of P, **(4)**
b) the direction of the acceleration of P, giving your answer as a bearing to the nearest degree. **(3)**

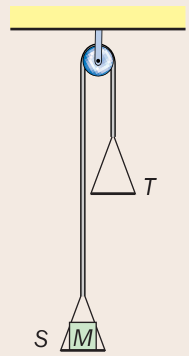
At time $t = 0$, P is at the point with position vector $(\mathbf{i} - 2\mathbf{j})$ m and is moving with velocity $-11\mathbf{i}$ m s^{-1}. Find, when $t = 2$,
c) the speed of P, **(4)**
d) the position vector of P. **(3)**

2 A cricket ball is thrown vertically upwards from ground level with speed u m s^{-1} and takes 4 s to reach the ground again. Neglecting air resistance,
a) draw a velocity–time graph to represent the motion of the ball during the first 4 s, **(2)**
b) find the maximum height of the ball above the ground. **(6)**

3 A vertical wall is 33 m high. A ball is thrown vertically upwards from a point on ground level close to the wall with an initial speed of 25.9 m s^{-1}. Find for how long the ball will be above the top of the wall. **(7)**

4 A particle of mass 2 kg is at rest on a rough horizontal plane. The coefficient of friction between the particle and the plane is $\frac{1}{4}$. A horizontal force of 18 N is applied to the body for 4 s and is then removed. Find
a) the speed of the particle after 4 s, **(5)**
b) the total distance travelled by the body in coming to rest. **(7)**

5



Two scale pans, S and T, each of mass 2 kg, are attached to the ends of a light inextensible string. The string passes over a small smooth fixed pulley and the two scale pans hang at rest. A mass M kg is placed inside S, as shown in the figure. Given that the acceleration of S downwards is $\frac{1}{2}g$, find
a) the tension in the string, **(3)**
b) the value of M, **(5)**
c) the magnitude of the normal reaction exerted on the mass by S. **(3)**

6 A non-uniform rod AB has mass $5m$ and length $3a$. The rod rests in equilibrium in a horizontal position on two supports at the points X and Y, where AX = XY = YB = a. A particle of mass $2m$ is fixed to the rod at B. Given that the rod is on the point of tilting about Y, find the distance of the centre of mass of the rod from B. **(5)**

7 A uniform rod AB of mass m and length $6a$ rests horizontally between two smooth pegs X and Y. Peg X is below the rod with AX = $4a$ and peg Y is above the rod with YB = a. A particle of mass $3m$ is hung from A.
a) A vertical force $\mathbf{P}$ acts downwards at B.
i) Draw a clear diagram showing all the external forces acting on the rod. **(2)**

Given that the rod is in equilibrium
ii) calculate the greatest possible value of the magnitude of $\mathbf{P}$. **(4)**

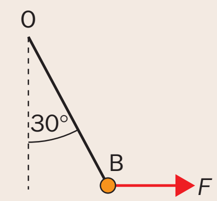
The force $\mathbf{P}$ is replaced by a vertical force $\mathbf{Q}$ acting upwards at B. The peg Y will break if the force on it exceeds $15mg$. Given that the rod is in equilibrium
b) calculate the greatest possible value of the magnitude of $\mathbf{Q}$ **(4)**

8 A particle is placed on a rough plane inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The particle is maintained in equilibrium by a horizontal force of magnitude 20 N which acts in the vertical plane containing the line of greatest slope of the inclined plane through the particle. The coefficient of friction between the particle and the plane is $\frac{1}{2}$. Given that the particle is on the point of slipping up the plane,

a) find the normal reaction of the plane on the particle, **(4)**

b) the weight of the particle. **(3)**

9



One end of a light inextensible string is attached to a fixed point O. The other end is attached to a small body B of weight 10 N. The body B is kept in equilibrium, with the string making an angle of 30° with the vertical, by a constant horizontal force of magnitude F as shown in the figure.

a) Find the magnitude, T, of the tension in the string. **(3)**

b) Show that $T = 2F$. **(3)**

10 A nail of mass 0.05 kg is driven horizontally into a fixed block of wood by a hammer of mass 1 kg. Immediately before striking the nail the hammer is moving with speed 10.5 m s^{-1}. Assuming that the hammer does not rebound

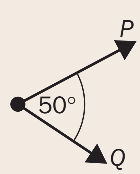
a) find the common speed of the nail and hammer immediately after the blow. **(3)**

The resistance to the nail when penetrating the block is modelled as being of constant magnitude R. Given that the nail is driven 0.07 cm into the block

b) find the value of R. **(5)**

Bank C

1



Two forces, **P** and **Q**, act on a particle. The magnitude of **P** is 6 N and the magnitude of **Q** is 4 N. The angle between the directions of **P** and **Q** is 50° as shown in the figure. The resultant of **P** and **Q** is **R**.

a) Find, to 3 significant figures, the magnitude of **R**. **(5)**

b) Find, in degrees to one decimal place, the angle between the direction of **R** and the direction of **Q**. **(4)**

2 A stone is dropped from the top of a building of height h and hits the ground t seconds later. Neglecting air resistance,

a) write down an equation relating h and t. **(2)**

One second later another stone is thrown vertically downwards from the top of the same building with a speed of 19.6 m s^{-1}. Given that the two stones strike the ground at the same time and neglecting air resistance,

b) write down another equation relating h and t. **(2)**

c) Find the height of the building. **(4)**

3 A train normally travels in a straight line at a constant speed of 40 m s^{-1}. However, repairs are being carried out to 1800 m of track between Q and R. Because of this the speed of the train has to be restricted to 16 m s^{-1} on this stretch of track. In order that the speed of the train is 16 m s^{-1} when it arrives at Q, the brakes are applied when the train is at the point P. The brakes produce a constant deceleration of 1.5 m s^{-2}. When the train reaches R it accelerates with constant acceleration so that it reaches its normal speed of 40 m s^{-1} at the point S where RS = 1344 m. Find

a) the time for which the brakes are applied, **(3)**

b) the distance PQ, **(3)**

c) the time for which the train accelerates, **(3)**

d) the total time lost by the train due to the speed restriction. **(4)**

4 A car of mass 900 kg tows a caravan of mass 600 kg along a straight horizontal road by means of a light rigid tow bar. The resistance to motion is proportional to mass. Given that the caravan experiences a resistance of 200 N

a) find the resistance experienced by the car. **(2)**

Given that the acceleration of the system is $\frac{2}{3}$ m s^{-2}, find

b) the tractive force provided by the engine of the car, **(3)**

c) the tension in the tow bar. **(3)**

When the speed of the car is 16 m s^{-1} the driver sees a hazard ahead and applies the brakes to bring the car to rest. Given that the overall braking force is 1500 N and that the resistances to the motion of the car and the caravan remain the same as before,

d) find the distance travelled by the car in coming to rest, **(5)**

e) determine the magnitude and nature of the force in the tow bar during the braking period. **(4)**

5 A buoy of mass 20 kg is held below the surface of the water by a vertical rope attached to the sea-bed. The upward buoyancy force from the water is 244 N. The rope breaks and the buoy rises up to the surface. In an initial model, the resistance to motion provided by the water is assumed to be of constant magnitude 12 N.

a) Find the upward acceleration of the buoy. **(3)**

In a refined model the magnitude of the resistance to motion provided by the water is assumed to be $3v^2$ N where v m s^{-1} is the speed of the buoy.

b) Find, under this model, the maximum speed of the buoy on its way to the surface. **(5)**

6 A window cleaner whose weight is 720 N sits on a light plank supported by means of two vertical ropes. The plank is horizontal and the distance between the ropes is 6 m.

a) Explain why the window cleaner must sit between the two ropes. **(4)**

b) Find the tension in each rope when the window cleaner sits 2 m from one of the ropes. **(4)**

7 A wooden beam AB rests on two supports, one at A and the other at B. The beam is in equilibrium and is horizontal. The beam is modelled as a non-uniform rod. Given that the support forces at A and B have magnitudes 120 N and 280 N respectively and that the length of the beam is 10 m find how far the centre of mass of the beam is from A. **(4)**

8 Two particles P and Q are attached to the ends of a light inextensible string. One of the particles P, of mass 0.13 kg, is placed on a rough plane inclined at an angle of 30° to the horizontal. The coefficient of friction between the particle and the plane is $\dfrac{1}{\sqrt{3}}$. The string passes over a small smooth pulley which is fixed at the top of the plane and supports the other particle Q, of mass m, which hangs freely. The system is at rest in equilibrium. Given that the string lies along a line of greatest slope of the plane and that P is on the point of slipping up the plane

a) find the magnitude of the normal reaction between P and the plane, **(2)**

b) the value of m. **(5)**

9 A body of mass M kg is supported in equilibrium by two ropes attached to it. One rope is inclined at 30° to the vertical and the other at 60° to the vertical. The body is modelled as a particle and the ropes are modelled as being light and inextensible. Given that either rope breaks if the tension in it exceeds 4900 N, find, to three significant figures, the greatest possible value of M. **(7)**

10 Two particles P and Q of masses $2m$ and m respectively are moving towards each other on a smooth horizontal plane and collide directly. Immediately *before* the collision the speed of P is u. Immediately *after* the collision, Q rebounds with speed u in the opposite direction. Given that the magnitude of the impulse in the collision is $4mu$, find

a) the speed and direction of motion of P immediately after the collision, **(3)**

b) the speed of Q immediately before the collision. **(3)**

Bank D

1 A particle P of mass 2.5 kg moves under the action of a single constant force $\mathbf{F}$. At time $t = 0$ the velocity of P is $(-2\mathbf{i} + \mathbf{j})$ m s^{-1} and 2 seconds later its velocity is $(4\mathbf{i} - 7\mathbf{j})$ m s^{-1}.

a) Find, in vector form, the acceleration of P. **(2)**

b) Find the magnitude of $\mathbf{F}$. **(2)**

c) Find, to the nearest degree, the acute angle between the line of action of $\mathbf{F}$ and the line $y = x$. **(4)**

2 [*In this question, the horizontal unit vectors* **i** *and* **j** *are directed due east and due north respectively.*]

The velocity of a particle P at time t seconds is modelled by the formula

$$\mathbf{v} = (3t^2 + 1)\mathbf{i} + (7t - 1)\mathbf{j} \text{ m s}^{-1}.$$

a) Find the direction of motion of P after 2 seconds. **(3)**

b) At what time is P first moving parallel to the vector $\mathbf{i} + \mathbf{j}$? **(6)**

3 Two sets of traffic lights on a straight horizontal road are 2145 m apart. A car takes two minutes to travel from one set to the other. It starts from rest from one set and accelerates uniformly for 30 s. It the moves with constant speed before uniformly decelerating to rest for the final 15 s.

a) Represent the motion on a speed–time graph. **(2)**

b) Find the acceleration of the car. **(5)**

4 The point A is on a rough plane which is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$. A particle P of mass 2 kg is projected from A up a line of greatest slope of the plane with speed 10 m s^{-1}. The coefficient of friction between P and the plane is $\frac{1}{2}$.

exam practice

a) Find the distance travelled by P before first coming to rest. **(7)**
b) Show that P will slide back down the plane. **(3)**
c) Find the speed of P as it passes through A. **(7)**

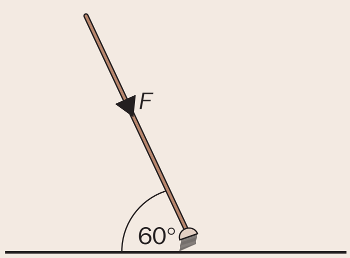
5 A car of mass 2000 kg tows a caravan of mass 800 kg up a slope which is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{7}{24}$. The engine of the car provides a driving force of constant magnitude 10 000 N and the total resistance on each of the car and caravan is 150 N per tonne.
a) Find the acceleration of the car. **(4)**

The tow bar is modelled as being light, rigid and inextensible.
b) Find the tension in the tow bar. **(3)**

6 A uniform beam AB of mass 20 kg and length 2.4 m is at rest in equilibrium in a horizontal position. The beam is supported by two vertical ropes XP and YQ attached to the beam at the points P and Q where AP = 0.4 m and QB = 0.6 m. Find
a) the tension in XP, **(4)**
b) the tension in YQ. **(2)**

7 A non-uniform rod ABC of length 12 m has its mass distributed such that the weight force acts downwards from a point G, where $AG = 5$ m. Three light strings are attached to the rod at A, B and C where $AB = 8$ m. The rod is at rest in equilibrium with the three strings vertical.
a) Given that the tension in the string attached at B has magnitude 25 g, find the magnitudes of the tensions in the other two strings. **(4)**
b) Find how far the point B would have to be moved in order for the magnitudes of the tensions in all three strings to be the same. **(4)**

8



A broom is being used to sweep a rough horizontal floor. A force of magnitude F newtons is applied downwards along the handle and the handle is kept at a constant angle of 60° to the horizontal as shown in the figure. The broom head moves along the floor with constant speed. The handle is modelled as a light rod and the broom head is modelled as a particle of mass 0.5 kg.
a) Explain why it is reasonable to ignore air resistance. **(1)**

Given that the coefficient of friction between the broom head and the floor is 0.5
b) find, to 3 significant figures, the value of F. **(7)**

9 A bullet of mass 100 g is fired into a wooden block of mass 4.9 kg which lies at rest on a rough horizontal floor. The bullet enters the block horizontally at 300 m s^{-1} and becomes embedded in it.
a) Find the initial speed of the block. **(3)**

The block slides along the floor eventually coming to rest. Given that the coefficient of friction between the floor and the block is 0.5
b) find, to 3 significant figures, how far the block slides. **(6)**

10 A particle P of mass $2m$ and a particle Q of mass $3m$ are attached to the ends of a light inextensible string. The particles are at rest and next to each other on a smooth horizontal plane with the string slack. Particle P is then projected along the plane directly away from Q with a speed $2u$.
a) Find the speed of Q immediately after the string goes taut. **(3)**
b) Explain how in your calculation you have used the modelling assumptions that the string is
i) light
ii) inextensible. **(2)**
c) Find the impulse transmitted through the string as the string goes taut. **(3)**

Answers

Page 7 End of Chapter 1

1 **a** modelling real-life situations
b Define, Model, Analyse, Interpret
c Kinematics, Dynamics, Statics
d particle
e rigid body
f particle
g inelastic
h modelling assumptions

2 Light

3 Lamina

4 A particle has no surface area.

5 Make/model of car, length, colour, number of passengers, number of doors, roof rack, spoiler fins.

6 Kinematics/moving, Statics/still, Dynamics/changing.

7 Parachuting, dropping a feather, aeroplane flying, hang-gliding, badminton, etc.

8 Lamina

9 Mass of book, height of window above ground, how book thrown.

10 Mass of barrel, angle of slope, length of road, how motion started.

Page 13

1 $|\mathbf{s}| + 31.0$ m (3 s.f.), 110.8° (1 d.p.)

2 17

3 $(9.15\mathbf{i} + 104.6\mathbf{j})$ m (3 s.f.)

4 $(1.55\mathbf{i} - 5.80\mathbf{j})$ m

Page 18 End of Chapter 2

1 **a** magnitude **b** direction
c bold **d** underlining
e Pythagoras' Theorem **f** trigonometry
g sine, cosine **h** parallel

2 scalar

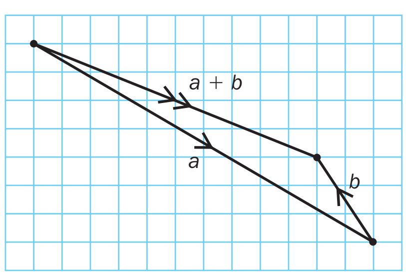
3 26 units

4 205.2° (1 d.p.)

5 $-5.02\mathbf{i} + 2.9\mathbf{j}$ (3 s.f.)

6 $\begin{pmatrix} -223 \\ 201 \end{pmatrix}$ m (3 s.f.)

7 $\mathbf{a} + \mathbf{b} = \begin{pmatrix} 10 \\ -4 \end{pmatrix}$



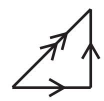
8 $\mathbf{0}$, the zero vector

9 $\begin{pmatrix} 5 \\ -1 \end{pmatrix}, \begin{pmatrix} -25 \\ 5 \end{pmatrix}, \begin{pmatrix} 2.5 \\ -0.5 \end{pmatrix}$ and $\begin{pmatrix} 30 \\ -6 \end{pmatrix}$

10 $9\mathbf{i} - 6\mathbf{j}$

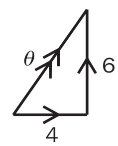
11 $17^2 = t^2 + 8^2 \rightarrow t = \pm 15$

12 **a**



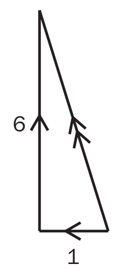
45°

b



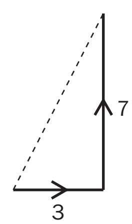
$\theta = \tan^{-1}(\frac{4}{6}) = 33.7°$ (1 d.p.)

c



$\theta = \tan^{-1}(\frac{1}{9}) = 6.3°$ (1 d.p.)

d



$\theta = \tan^{-1}(\frac{3}{7}) = 23.2°$ (1 d.p.)

13 **a** $30 \times \frac{1}{5}(3\mathbf{i} = 4\mathbf{j}) = 18\mathbf{i} - 24\mathbf{j}$
b $5 \times \frac{1}{25}(7\mathbf{i} = 24\mathbf{j}) = \frac{7}{5}\mathbf{i} - \frac{24}{5}\mathbf{j}$

14 **a** $\frac{1}{10}(6\mathbf{i} + 8\mathbf{j}) = 0.6\mathbf{i} + 0.8\mathbf{j}$
b $\frac{1}{25}(7\mathbf{i} - 24\mathbf{j}) = \frac{7}{25}\mathbf{i} - \frac{24}{25}\mathbf{j}$
c $\frac{1}{\sqrt{2}}(\mathbf{i} - \mathbf{j}) = \frac{1}{\sqrt{2}}\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{j}$
d $\frac{1}{\sqrt{40}}(-2\mathbf{i} + 6\mathbf{j}) = \frac{-2}{\sqrt{40}}\mathbf{i} + \frac{6}{\sqrt{40}}\mathbf{j}$

15 $\frac{12}{3} = \frac{-4}{b} \Rightarrow b = -1$

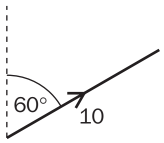
16 **a** $\mathbf{i} - 3\mathbf{j} + a(2\mathbf{i} + \mathbf{j}) = (1 + 2a)\mathbf{i} + (-3 + a)\mathbf{j}$
So, $-3 + a = 0 \Rightarrow a = 3$
b $b(\mathbf{i} - 3\mathbf{j}) + 2\mathbf{i} + \mathbf{j} = (b + 2)\mathbf{i} + (-3b + 1)\mathbf{j}$
So, $b + 2 = 0 \Rightarrow b = -2$

17 $\frac{2t - 5}{1} = \frac{-(4 - t)}{-1} \Rightarrow t = 3$

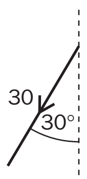
18 $\mathbf{v}_A \parallel \mathbf{v}_B \Rightarrow \dfrac{t-2}{1} = \dfrac{2t-3}{t}$

$\Rightarrow t^2 - 2t = 2t - 3$
$\Rightarrow t^2 - 4t + 3 = 0$
$\Rightarrow (t-3)(t-1) = 0$
$\Rightarrow t = 3 \text{ or } 1$

19 a



$10 \sin 10 i + 10 \cos 60° j$
$= 5\sqrt{3} i + 5j$

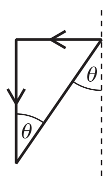


$-39 \sin 30° i - 30 \cos 30° j$
$= -15i - 15\sqrt{3} j$

$\therefore \mathbf{R} = (5\sqrt{3} - 15)i + (5 - 15\sqrt{3})j$
$= 5(\sqrt{3} - 3)i + 5(1 - 3\sqrt{3})j$

b $|\mathbf{R}| = 5\sqrt{(\sqrt{3}-3)^2 + (1-3\sqrt{3})^2}$
$= 5\sqrt{3 - 6\sqrt{3} + 9 + 1 - 6\sqrt{3} + 27}$
$= 5\sqrt{40 - 12\sqrt{3}}$
$= 21.9 \text{ (1 d.p.)}$

$\mathbf{R} = -5(3 - \sqrt{3})i - 5(3\sqrt{3} - 1)j$

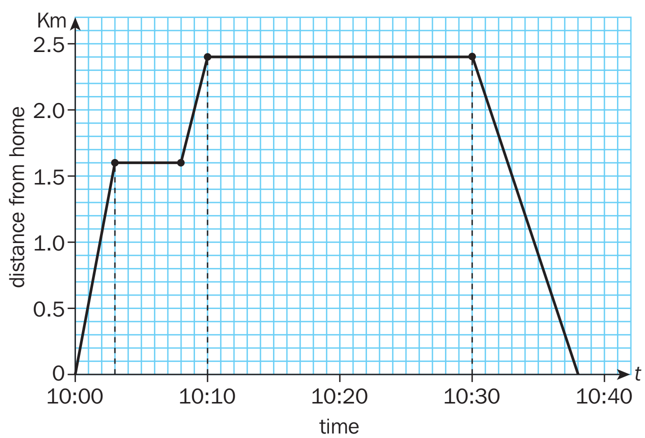


$\tan \theta = \dfrac{\cancel{5}(3-\sqrt{3})}{\cancel{5}(3\sqrt{3}-1)} = \dfrac{3-\sqrt{3}}{3\sqrt{3}-1}$

$\therefore$ Bearing is $180° + \theta$
$= 196.8 \simeq 197° \text{ (3 s.f.)}$

Page 25

1



0.533 km per minute (3 s.f.)
0.4 km per minute
0.3 km per minute

2 The bucket is raised to the top of the building at a constant speed.
It remains there for 30 seconds.
It is lowered at a constant speed to ground level.
After 12 seconds, this motion is repeated.
Upwards speed 0.25 ms^{-1}
downwards speed 0.6 ms^{-1}

Page 29 End of Chapter 3

1 **a** time **b** distance
c velocity **d** acceleration
e displacement **f** multiplying the units on the axes

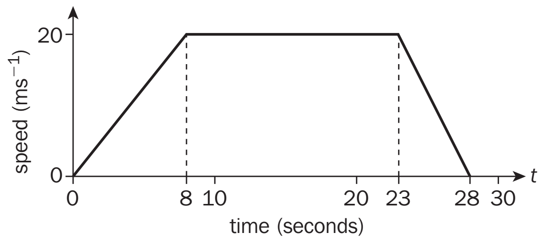
2 object is stationary

3 6.47 ms^{-1} (3 s.f.)

4 160 000 m in 8100 s; 19.8 ms^{-1} (3 s.f.)

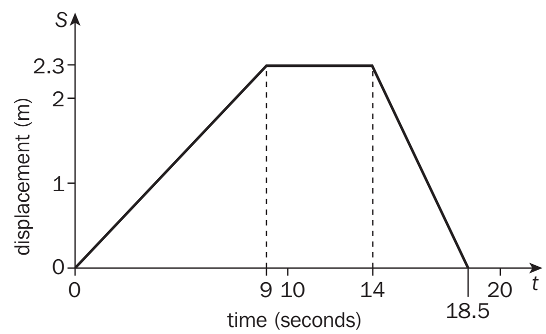
5 acceleration is zero, i.e. constant velocity

6



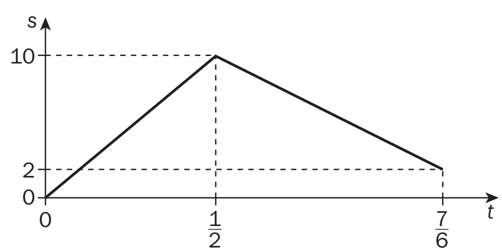
car has travelled 430 m

7



descent velocity 0.511 ms^{-1} (3 s.f.)

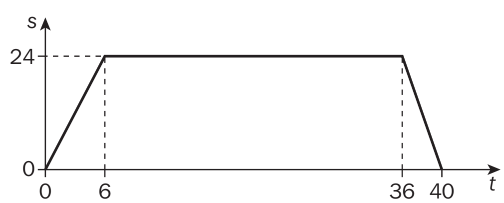
8 **a**



b Av. speed $= \dfrac{18}{(7/6)} = \dfrac{108}{7} = 15\frac{3}{7} \text{ ms}^{-1}$

c Av. vel. $= \dfrac{2}{(7/6)} = \dfrac{12}{7} = 1\frac{5}{7} \text{ ms}^{-1}$

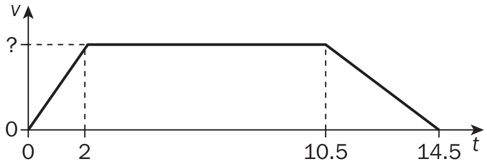
9 **a**



b Distance $= \dfrac{(40+30)}{2} .24 = 840 \text{ m}$

c Av. speed $= \dfrac{840}{40} = 21 \text{ ms}^{-1}$

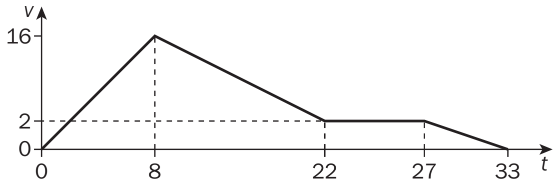
10 a



b $100 = \frac{(10.5 + 8.5)}{2}.V \to V = \frac{200}{19} = 10.5\ \text{ms}^{-1}$ (1 d.p.)

c Total distance $= 100 + \frac{1}{2}.4V = 121.1$ m (1 d.p.)

11 a



b Final deceleration $= \frac{2}{6} = \frac{1}{3}\ \text{ms}^{-2}$

c Total distance $= (4 \times 16) + 14 \times \frac{(16 + 2)}{2} + (5 \times 2) + (3 \times 2)$

$= 64 + 126 + 10 + 6$
$= 206$ m

Page 39 End of Chapter 4

1 **a** constant or uniform **b** SI
c $t = \ldots$ **d** list
e $v^2 = u^2 + 2as$ **f** t

2 540 m

3 $68\ \text{ms}^{-1}$, $-68\ \text{ms}^{-1}$

4 $t = \frac{v - u}{a}$

5 $6\frac{2}{3}\ \text{ms}^{-1}$

6 $s = vt - \frac{1}{2}at^2$

7 1200 m

8 $0.778\ \text{ms}^{-2}$

9 $\begin{pmatrix} -22\frac{1}{2} \\ 135 \end{pmatrix}$ m 137 m (3 s.f.)

10 $t = 10$ s, $a = (-2\mathbf{i} + 3\mathbf{j})\ \text{ms}^{-2}$

11 $s = \frac{(3 + 7)}{2}.12 = 60$ m

12 a $0^2 = 60^2 - 2a \times 1200$

$\Rightarrow a = \frac{3600}{2400} = 1.5\ \text{ms}^{-2}$

b $0 = 60 - 1.5t \Rightarrow t = 40$ s

13 a $\frac{54\ \text{km}}{1\ \text{h}} = \frac{54\cancel{00}}{36\cancel{00}} = 15\ \text{ms}^{-1}$

b 72 kph $= 20\ \text{m}^{-1}$

so, $a = \frac{20 - 15}{10} = \frac{1}{2}\ \text{ms}^{-2}$

c $s = \frac{(15 + 20)}{2}.10 = 175$ m

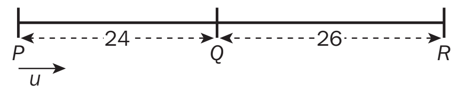
14 a $5 = 7T - \frac{1}{2}.4T^2 \Rightarrow 2T^2 - 7T + 5 = 0$

b $(2T - 5)(T - 1) = 0 \Rightarrow T = 1$ or $\frac{5}{2}$

$\therefore$ Time is 2 seconds (from 0–1 sec and from 2.5–3.5 sec)

15 $t : 0$ $t = 3$ $t = 5$

a



$P \to Q$: $24 = 3u + \frac{1}{2}ax3^2$
$\Rightarrow 16 = 2u + 3a$ ①
$P \to R$: $50 = 5u + \frac{1}{2}ax5^2$
$\Rightarrow 20 = 2u + 5a$ ②
② − ①: $4 = 20 \Rightarrow a = 2\ \text{ms}^{-2}$

b $u = 5\ \text{ms}^{-1}$

16 a $729 = \frac{1}{2} \times a \times 90^2 \Rightarrow a = 0.18\ \text{ms}^{-2}$

b $s = 729 + 600v$ where $v = 0.18 \times 90$
$= 16.2\ \text{ms}^{-1}$

$= 10\,449$ m

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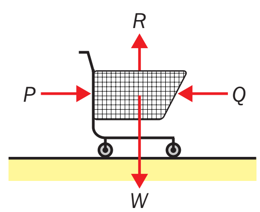
1 9310 N

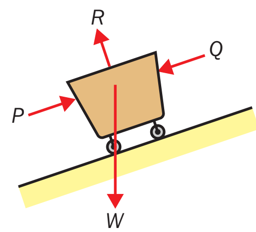
2 0.8624 N

3 Shuttle cock (and tennis ball)

4 **a**



b



$P = Q$
$R = W$

5 $\mathbf{F} = \mathbf{S} + \mathbf{T} + \mathbf{Q}$
$\mathbf{R} = \mathbf{P} + \mathbf{W}$

6 $a = 120$ N, $b = 347$ N (3 s.f.)

7 $-22.2m\mathbf{i} + 43.7m\mathbf{j}$

8 $R = 339$ N (3 s.f.)
$T = 196$ N

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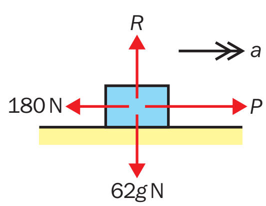
1 thrust

2 $1040\ \text{ms}^{-2}$

3 50 N

4 $2.30\ \text{ms}^{-2}$ (3 s.f.) up the plane

5



a 180 N
b 195.5 N

6 Throwing the javelin forward will cause him to roll backwards on his skates.
He could rapidly move one arm or leg backwards as his throwing arm moves forwards.

7 $R = 8090$ N (3 s.f.)
$a = 0.372$ ms^{-1} (3 s.f.)

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1 **a** the centre of the Earth **b** the Normal Reaction force
c mass, g **d** motion
e balance/equate **f** $\mathbf{F} = m\mathbf{a}$
g parallel **h** equal and opposite pairs

2 0.051 N

3 $(333\mathbf{i} - 411\mathbf{j})$ N (3 s.f.)

4 **a** P = 50 N, R = 20 N
b R = 110 N, Q = 315 N
c R = 305 N, T = 110 N

5 4.36 ms^{-2} (3 s.f.)

6 **a** $12 = 6a \Rightarrow a = 2$ ms^{-2}
b $v = 3 \times 2 = 6$ ms^{-1}

7 $8 = 0.4a \Rightarrow a = 20$
$4 = 12 - 20t = 0.4 \Rightarrow t = \frac{8}{20}$

8 **a** $15 = 5a \Rightarrow a = 3$ ms^{-2}
b $F^2 = 15^2 + 8^2 - 2 \times 15 \times 8 \times -\frac{1}{2}$
$= 289 + 120 = 409 \Rightarrow F = \sqrt{409}$
$\therefore\ a = \dfrac{\sqrt{409}}{5} = 4.04$ ms^{-2} (3 s.f.)
c $F^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \cos 70°$
$= 52 - 48 \cos 70° = 35.58$
$F = 5.965\ \therefore\ a = 1.19$ ms^{-2} (3 s.f.)

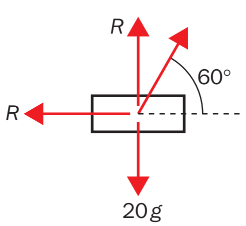
9 **a** $2\mathbf{i} - 4\mathbf{j}$
b $|a| = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$ ms^{-2}

10 **a** $a = \frac{1}{6}(18\mathbf{i} - 12\mathbf{j}) = 3\mathbf{i} - 2\mathbf{j}$
b $v = (2\mathbf{i} + 4\mathbf{j}) + 4(3\mathbf{i} - 2\mathbf{j})$
$= 14\mathbf{i} - 7\mathbf{j}$
$|v| = \sqrt{14^2 + 7^2} = \sqrt{245} = 7\sqrt{5}$ ms^{-1}

11 $a = 2.5 \times \frac{1}{5}(3\mathbf{i} - 4\mathbf{j}) = \frac{3}{2}\mathbf{i} - 2\mathbf{j}$
So, $(a\mathbf{i} + 3\mathbf{j}) + (-\mathbf{i} - 17\mathbf{j}) + (12\mathbf{i} + 6\mathbf{j}) = 0.2(\frac{3}{2}\mathbf{i} - 2\mathbf{j})$
$\Rightarrow a - 1 + 12 = 0.3 \Rightarrow a = -10.7$
$\Rightarrow 3 - 17 + b = 0.4 \Rightarrow b = 14.4$

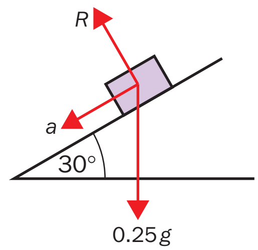
12 $T = 10 \times 0.2 = 2$ N

13



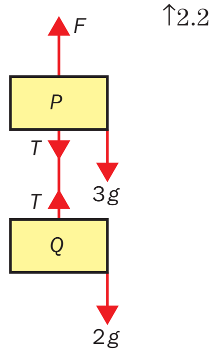
$R(\rightarrow)$, $60 \cos 60° - F = 0$
$\Rightarrow F = 30$ N

14



$R(\leftarrow)$
$\cancel{0.25}\,g \cos 60° = \cancel{0.25}a$
4.9 ms$^{-2} = a$

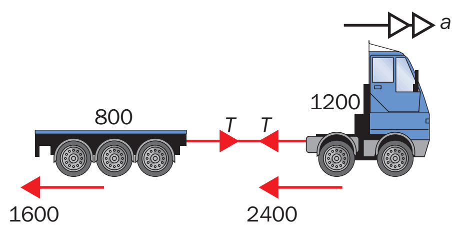
15 **a**



$P(\uparrow)$, for whole system,
$F - 5\,g = 5 \times 2.2 = 11$
$\Rightarrow F = 60$ N

b *Fa Q*: $P(\uparrow)$, $T - 2\,g = 2 \times 2.2 = 4.4$
$\Rightarrow$ T = 24 N

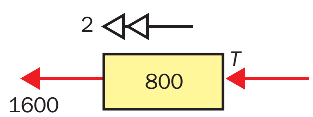
16 **a**



$R(\rightarrow)$, $10\,000 - 1600 - 2400 = 2000a$
3 ms$^{-2} = a$

b For trailer: $R(\rightarrow)$, $T - 1600 = 800 \times 3$
$\Rightarrow T = 4000$ N

c $R(\rightarrow)$, $-1600 - 2400 = 2000a$
$\Rightarrow a = -2$ ms^{-2}
i.e. deceleration is 2 ms^{-2}



$R(\leftarrow)$, $1600 + T = 1600$
$T = 0$ N

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1 **a** Ns (newton seconds) **b** impulse
c mass, velocity **d** constant
e external forces acting **f** directly

2 coalesce

3 3360 Ns

4 0.0816 Ns

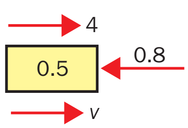
5 13.5 Ns

6 **a** 0.01054 Ns **b** 0.120 N (3 s.f.)

7 0.55 ms^{-1}

8 0.66 ms^{-1}

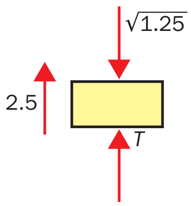
9



$-0.8 = 0.5\,(v - 4)$
$-1.6 = v - 4$
$\Rightarrow v = 2.4\ \text{ms}^{-1}$

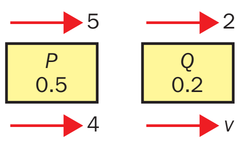
10 **a** $v^2 = 2g \times 0.6 \Rightarrow v = \sqrt{1.2g} = 3.4\ \text{ms}^{-1}$ (2 s.f.)

b



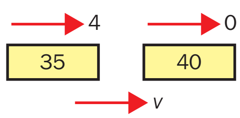
$I = 2(2.5 - -\sqrt{1.2g}) = 12$ Ns (2 s.f.)

11



$(5 \times 5) + (2 \times 0.2) = (4 \times 0.5) + 0.2v$
$2.9 = 2 + 0.2v$
$0.9 = 0.20$
$v = 4.5\ \text{ms}^{-1}$

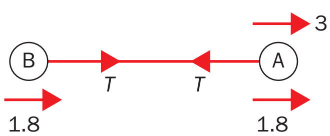
12 **a**



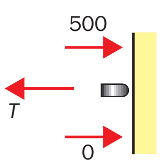
CLM: $4 \times 35 = 75v$
$140 = 75v$
$v = \frac{140}{75} = \frac{28}{15}\ \text{ms}^{-1}$

$T = 1000 \times 40 \times \frac{28}{15} = 1000 \times 8 \times \frac{28}{3} = \frac{224\,000}{3}$ Ns
$= \frac{224}{3}$ kNs

13 **a** CLM: $9 = 5u \Rightarrow u = 1.8\ \text{ms}^{-1}$
b For B: $T = 2 \times 1.8 = 3.6$ Ns



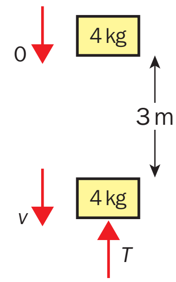
14 **a**



$\rightarrow$ +ve
$-T = 0.02\,(0 - 500)$
$T = 10$ Ns

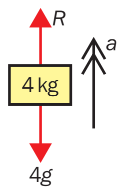
b $F = \frac{10}{0.01} = 1000$ N

15 **a**



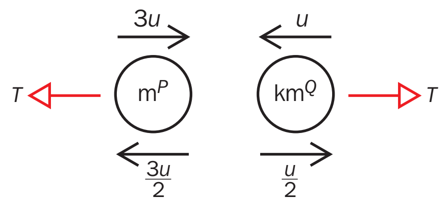
$v^2 = 2 \times 3 \times 9.8 = 6\,g$
$v = 7.668\ldots$
$-T = 4(0 - v)$
$T = 4v = 30.7$ Ns (3 s.f.)

b



$R - 4g = 4a$
$R = 4g + 4a$
$v^2 = u^2 + 2as$
$0^2 = 6g - 2a \times \frac{2}{100}$
$\Rightarrow a = \frac{600}{4}g$
$= 150\,g$
$\therefore\ R = 4\,g + 600\,g = 604\,g = 5920$ N (3 s.f.)

16



a CLM ($\rightarrow$): $3\cancel{mu} - k\cancel{mu} = -\frac{3\cancel{mu}}{2} + \frac{k\cancel{mu}}{2}$

$6 - 2k = -3 + k$
$9 = 3k$
$3 = k$

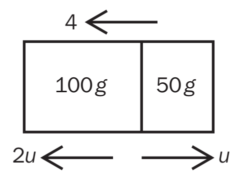
b For Q: ($\rightarrow$) $T = 3m\left(\frac{u}{2} - -u\right)$

$= 3m - \frac{3u}{2} = \frac{9mu}{2}$

$\left[\text{or} \mid \text{For } P\text{: } (\rightarrow)\ \ -T = m\left(-\frac{3u}{2} - 3u\right)\right]$

$-T = -\frac{9mu}{2}$

17



CLM:
$150 \times 4 = (100 \times 2u) - (50 \times u)$
$600 = 150u$
$4 = u$
$\therefore\ u = 4$

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1 **a** surfaces **b** prevent
c limiting **d** smooth
e normal reaction **f** coefficient
g motion **h** the direction of motion

2 speed, mass

3 0

4 14.4 N

5 0.358 (3 s.f.)

6 300 N

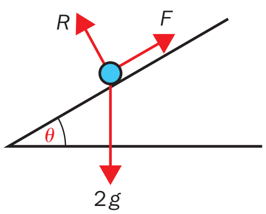
7 $F_{max} = 23.52$ N, tin will not move

8 48.6 N (3 s.f.)

9 **i** **a** $R = 8\,g \quad \therefore\ F_{max} = \frac{5}{7} \times 8\,g = 56$ N
$\therefore$ Friction is 56 N
b $a = 0$
ii **a** $R = 10\,g \quad \therefore\ F_{max} = \frac{2}{5} \times 10\,g = 4\,g = 39.2$ N
$\therefore$ Friction is 39.2 N
b $0.8 = 10a \Rightarrow a = 0.08\ \text{ms}^{-2}$
iii **a** $R = 5\,g \quad \therefore\ F_{max} = \frac{4}{7} \times 5\,g = 28$ N
$\therefore$ Friction is 25 N
b $a = 0$
iv **a** $R = 10\,g \quad \therefore\ F_{max} = \frac{1}{2} \times 10\,g = 49$ N
$\therefore$ Friction is 49 N
b $a = 0$

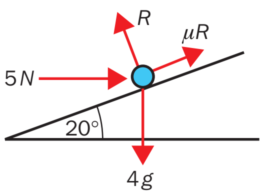
10 **a** $F = \mu R = \frac{1}{3} \times 6\,g = 2\,g = 19.6\ \text{N} = \text{P}$
b $\text{P} \sin 60° + R = 10\,g$
$\text{P} \cos 60° = \sqrt{3}R$
so, $\dfrac{\text{P}\sqrt{3}}{2} + \dfrac{\text{P}}{2\sqrt{3}} = 105$
$3\text{P} + \text{P} = 20\,g\sqrt{3}$
$\text{P} = 5\,g\sqrt{3} = 49\sqrt{3}$ N

11



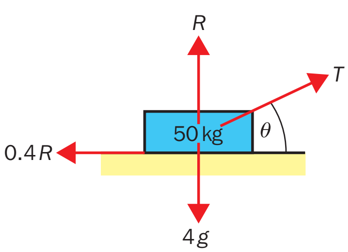
$R = 2\,g \cos\theta$
$F = 2\,g \sin\theta$
$\therefore\ \dfrac{F}{R} = \tan\theta$
$\therefore\ \tan\theta \leqslant \mu$
so, $\tan\theta \leqslant \frac{1}{2}$
$\therefore\ \theta = 26.6°$ (3 s.f.)

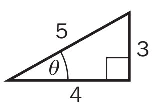
12



$(\nwarrow)\ R = 4\,g \cos 20° + 5 \sin 20°$
$(\nearrow)\ 5 \cos 20° - 4\,g \sin 20° + \mu\,(4\,g \cos 20° + 5 \sin 20°) = 0$
$5 - 4\,g \tan 20° + \mu\,(4\,g + 5 \tan 20°) = 0$
$\Rightarrow \mu = \dfrac{-5 + 4\,g \tan 20°}{4\,g + 5 \tan 20°} = 0.223$ (3 s.f.)

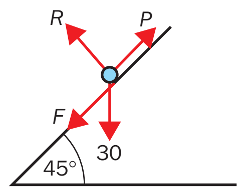
13





$\dfrac{4T}{5} = 0.4R \Rightarrow T = 0.5R$
$R + \dfrac{3T}{5} = 50\,g$
$\therefore\ 1.3R = 490$
$R = 380$ N (2 s.f.)
$T = 188$ N (3 s.f.)
$= 190$ N (2 s.f.)

14



$\mu = \frac{1}{3}$
$R = 30 \cos 45°$
$P = 30 \sin 45 + 10 \cos 45°$
$= 28.28$ N
$= 28.3$ N (3 s.f.)

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1 **a** turn
b distance of the line of action from the pivot
c clockwise, anticlockwise
d zero
e finding the total moment (or taking moments)
f trigonometry
g the total anticlockwise moment

2 fulcrum

3 5.7 Nm

4 72 kg

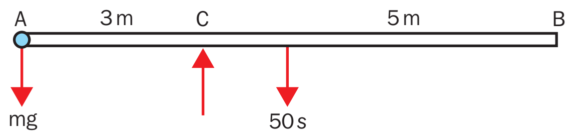
5 19.2 m (3 s.f.)

6 3.58 Nm clockwise

7 2 m from the other end

8 **a** $50 \times 4 = 2P \Rightarrow P = 100\ \text{N} \Rightarrow Q = 50$ N
b $10 \times 3 = 2R \Rightarrow R = 15\ \text{N} \Rightarrow S = 5$ N
c $420 = 14U \Rightarrow U = 30\ \text{N} \Rightarrow T = 40$ N
d $3X = 1500 \Rightarrow X = 500\ \text{N} \Rightarrow W = 200$ N

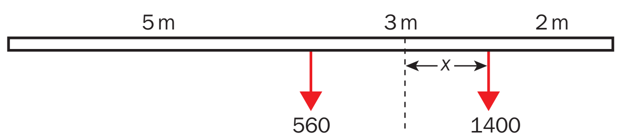
9



a $3\,m\cancel{g} = 50\,\cancel{g}$
$\Rightarrow m = \frac{50}{3}$
b $X = 50\,g\,(1 + \frac{1}{3})$
$= \dfrac{200\,g}{3}$

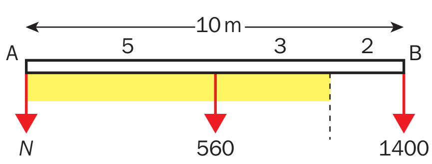
10 **a** $(\uparrow)\ P = 160$ N
$100 \times 2.5 + 50 \times (2.5 + 4x) = 160(2.5 + x)$
$250 + 125 + 200x = 400 + 160x$
$40x = 25$
$x = \frac{25}{40} = \frac{5}{8} = 0.6125$ m
b $Q = 100$ N
$200 = 40(2 + x)$
$x = 3$ m

11



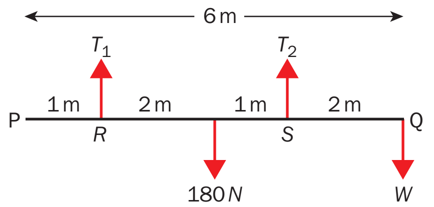
a $\overset{4}{\cancel{560}} \times 3 = \overset{10}{\cancel{1400}}x$
$\dfrac{12}{10} = x \quad \therefore \Rightarrow x = 1.2$ m
$\therefore$ From B, 0.8 m

b



$2800 = 8W + 560 \times 3$
$\Rightarrow W = 140\text{ N}$

12



a $M(R)$, $3T_2 = 2 \times 180$
$T_2 = 120\text{ N}$
So, $T_1 = 60\text{ N}$

b 0

c $180 \times 3 = 2T_2 \Rightarrow T_2 = 270\text{ N}$

d $W = 90\text{ N}$

Page 96 End of Chapter 9

1 **a** static, dynamic **b** statics
c coplanar **d** two
e resultant **f** component
g moments **h** triangle

2 Weight and Reaction

3 They must act in the same line

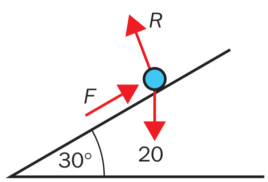
4 $\mathbf{F}_3 = (-13\mathbf{i} - 32\mathbf{j})\text{ N}$

5 $\mathbf{F}_3 = (12\mathbf{i} + 5\mathbf{j})\text{ N}$,
$|\mathbf{F}_3| = 13\text{ N}, 22.6°$

6 670 N (3 s.f.)

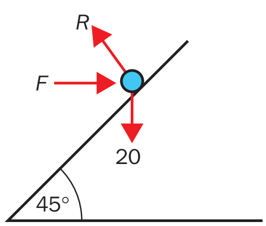
7 $|\mathbf{Q}| = 6\text{ N}$
$d = 1.83\text{ m}$ (3 s.f.)

8 **a**



$F = 20\cos 60° = 10\text{ N}$

b

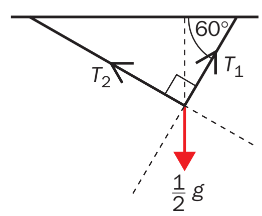


$F \cancel{\cos 45°} = 20 \cancel{\cos 45°}$
$F = 20\text{ N}$

9 **a** $(4\mathbf{i} - 5\mathbf{j}) + (16\mathbf{i} + 3\mathbf{j}) + (p\mathbf{i} + 8\mathbf{j}) = \mathbf{R}$
$\Rightarrow \mathbf{R} = (20 + p)\mathbf{i} + 6\mathbf{j} \quad || \quad 3\mathbf{i} + \mathbf{j}$
$\Rightarrow p = -2$

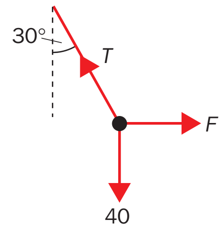
b $\mathbf{F} = -18\mathbf{i} - 6\mathbf{j}$
$\therefore |\mathbf{F}| = 6\sqrt{3^2 + 1^2} = 6\sqrt{10}\text{ N}$ (19.0 N) (3 s.f.)

10



$T_1 = 4.9\cos 30° = 4.24\text{ N}$ (3 s.f.)
$T_2 = 4.9\cos 60° = 2.45\text{ N}$

11 **a**



$T\cos 30° = 40 \Rightarrow T = \dfrac{80}{\sqrt{3}} = 46.2\text{ N}$ (3 s.f.)

$F = T\sin 30° = 30° = 23.1\text{ N}$ (3 s.f.)

12 $(3\mathbf{i} - \mathbf{j}) + a(-2\mathbf{i} + 3\mathbf{j}) + b(\mathbf{i} - 2\mathbf{j}) = \mathbf{O}$
$\Rightarrow (3 - 2a + b)\mathbf{i} + (-1 + 3a - 2b)\mathbf{j} = \mathbf{O}$
$\Rightarrow \quad 2a = b = 3$
$3a - 2b = 1$
$4a = 2b = 6$
$a = 5$
$b = 7$
$\mathbf{Q} = (-10\mathbf{i} + 15\mathbf{j})\text{ N}$
$\mathbf{R} = (7\mathbf{i} - 14\mathbf{j})\text{ N}$

Page 98 Revision Bank A

1 **a** $\theta = 71.6°$ **b** $|R| = 10\text{ N}$

2 **a** 25 km
b $r_C = (-9i + 6j) + t(3i + 12j)$
$r_D = (16i + 6j) + t(-9i + 3j)$
c $\overrightarrow{CD} = r_D - r_C = 25i + t(-12i - 9j)$
d Time is 5.05 pm

3 **a** 480 m **b** $t = 12$

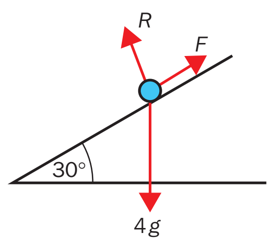
4 **a** $a = \frac{1}{2}\text{ ms}^{-2}$ **b** $\mu = 0.62$ (2 d.p.)
c A does reach the pulley

5 **a** 40 N (2 s.f.) **b** 6.2 m^{-2} (2 s.f.)

6 **a** 0.8 g **b** 1.2 g
c $M = 6$ **d** $x = \frac{40}{3}$

7 **a** $20\sqrt{3}\text{ N}$ **b** $10\sqrt{3}\text{ N}$

8 **a**



$R(\uparrow)$, $R = 4\text{ g}\cos 30° = 2\text{ g}\sqrt{3}$
$\therefore F_{MAX} = 0.2 \times 2\text{ g}\sqrt{3}$
$= 0.4\text{ g}\sqrt{3}$
component of weight down plane $= 4\text{ g}\cos 60°$
$= 2\text{ g}$
Since $2\text{ g}\sqrt{3} > 0.4\text{ g}\sqrt{3}$, it will slide downplane

b 13 N (2 s.f.)

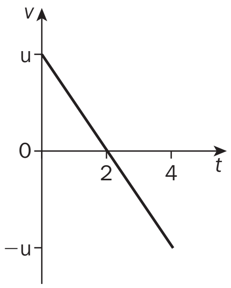
9 **a** 600 Ns **b** $v = 1\ \text{ms}^{-1}$ **c** $\mu = 0.5$ (1 d.p.)

10 **a** $1 > k$ **b** $3mu$

Page 99 Revision Bank B

1 **a** $10\ \text{ms}^{-2}$ **b** Bearing = 127°
c $13\ \text{ms}^{-1}$ **d** $-5\mathbf{i} - 14\mathbf{j}$

2 **a**



b $h = 19.6$ m

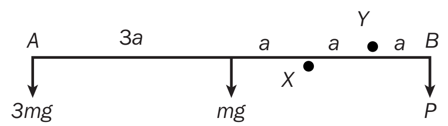
3 1 second

4 $26.2\ \text{ms}^{-1}$ **b** 193 m (3 s.f.)

5 **a** 3g **b** 4 **c** 2g

6 $\dfrac{7a}{5}$

7 **a** **i**



ii $\dfrac{13ms}{2} = \mathbf{P}$

b $\mathbf{Q} = mg$

8 **a** 20 N **b** 10 N

9 **a** $T = \dfrac{20\sqrt{3}}{3}$ N **b** R(→), $T \cos 60° = F$
$\Rightarrow T = 2F$

10 **a** Common speed is $10\ \text{ms}^{-1}$

b $R = \dfrac{1\,000\,000}{14} \times 1.05 = 75\,000$ N

Page 100 Revision Bank C

1 **a** $R = 9.10$ N **b** $\theta = 30.3°$ (1 d.p.)

2 **a** $h = \frac{1}{2}gt^2$ **b** $h = 2g(t-1) + \frac{1}{2}g(t-1)^2$
c 11.025 m

3 **a** 16 s **b** 448 m **c** 48 s **d** 86.7 s

4 **a** 300 N **b** 1500 N **c** 600 N
d 96 m **e** 600 N (thrust)

5 **a** $1.8\ \text{ms}^{-2}$ **b** $4\ \text{ms}^{-1}$

6 **a** If the window cleaner sat at point P, not between the ropes, then taking moments about P would mean that both tensions would be turning the *same* way and so equilibrium would be impossible.
b $T_2 = 240$ N
$T_1 = 480$ N

7 7 m

8 **a** 1.10 N (3 s.f.) **b** $m = 0.13$ kg

9 577 (3 s.f.)

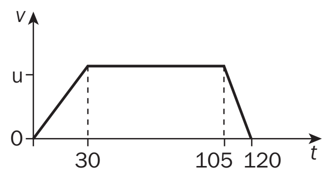
10 **a** u **b** $3u$

Page 101 Revision Bank D

1 **a** $(3\mathbf{i} - 4\mathbf{j})\ \text{ms}^{-2}$ **b** 12.5 N **c** 98°

2 **a** Direction is NE **b** $t = \frac{1}{3}$ is first time

3 **a**



b $\frac{11}{15}\ \text{ms}^{-2}$

4 **a** $\dfrac{100}{2g}$ **b** $2\text{ g} \sin \alpha > \dfrac{4g}{5}$
$\Rightarrow$ will slide down

c $2\sqrt{5}\ \text{ms}^{-1}$

5 **a** $0.68\ \text{ms}^{-2}$ (2 d.p.) **b** 2860 (3 s.f.)

6 **a** 112 N **b** 84 N

7 **a** $\dfrac{25\text{ g}}{3}$ $\dfrac{80\text{ g}}{3}$ **b** B moves 5 m towards A

8 **a** The broom is moving slowly
b $F = 37$ N (2 s.f.)

9 **a** $v = 6\ \text{ms}^{-1}$ **b** $s = 3.67$ m

10 **a** $V = \dfrac{4u}{5}$

b **i** Ignored the momentum of the string
ii Assumed both particles have the *same* speed immediately after the string goes taut

c $\dfrac{12mu}{5}$

Glossary

Acceleration: the rate of change of velocity with time. This is equal to the gradient of a velocity–time graph.

Acceleration–time graph; area under: gives the change in speed.

Acceleration, uniform: the size or magnitude of the acceleration must be constant and the direction must not change.

Area, of a trapezium: is found using the formula $A = \frac{1}{2}(a + b)h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between them.

Average speed: see Speed, average.

Average velocity: see Velocity, average.

Centre of mass: the position at which the total mass of a body can be taken as acting.

Coalesce, objects which: two objects which as a result of a collision stick together and move as one body afterwards.

Coefficient of friction: see Friction, coefficient of.

Collision: impact between two objects.

Commutative: an operation is commutative if the result is independent of the order of operation. For example, multiplication is commutative because the order does not affect the outcome, $3 \times 4 = 4 \times 3 = 12$.

Components: if the unit vectors in the direction of the x and y axes are denoted by **i** and **j**, then any vector can be written in the form:

$$x\mathbf{i} + y\mathbf{j} \quad \text{or} \quad \begin{pmatrix} x \\ y \end{pmatrix}$$

Three dimensional vectors will include a component in the direction of the z axis. This will be written in the form:

$$x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \quad \text{or} \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Concurrent forces: these are forces which act on the same point.

Connected particles: a simple system of two (or more) particles connected by a rod or string. They share the same acceleration if the string is taut.

Conservation of linear momentum: see Principle of conservation of linear momentum.

Coplanar: when points are all located on the same plane or surface, they are said to be coplanar. A system of forces all acting on the same plane is also referred to as coplanar.

Cosine rule: is a rule for calculating the length of a side or an angle in a triangle with no right angle:

$$a^2 = b^2 + c^2 - 2bc \cos \text{A} \quad \text{or} \quad \cos \text{A} = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = c^2 + a^2 - 2ca \cos \text{B} \quad \text{or} \quad \cos \text{B} = \frac{c^2 + a^2 - b^2}{2ca}$$

$$c^2 = a^2 + b^2 - 2ab \cos \text{C} \quad \text{or} \quad \cos \text{C} = \frac{a^2 + b^2 - c^2}{2ab}$$

where a, b and c are the lengths of the sides and A, B and C are the angles opposite these sides.

Coulomb, Charles Augustin de (1736–1806): 18th century French physicist and military engineer. Although best known for his work on electricity, he also developed the accepted principles of friction.

Deceleration (a.k.a. retardation): negative acceleration.

Density: the ratio of mass to volume. For laminas, density is the ratio of mass to area.

Dimensional analysis: the examination of units in a formula, to indicate whether the formula is correct. The units are given in terms of M, L, T (mass, length, time).

Direction: two vectors in the same direction will be parallel.

Displacement: this is a vector quantity and it is the distance travelled by a moving body in a specified direction. When a body returns to its original position, the displacement will be zero.

Displacement–time graph; gradient of: gives the velocity.

Distance: this is a scalar quantity and is given by the formula:

distance = speed × time (when speed is constant)

Distance–time graph; gradient of: gives the speed.

Dynamic equilibrium: occurs when an object is moving with constant velocity.

Dynamics: the modelling of motion using forces.

Equilibrium: occurs where there is no change in motion (see Static and Dynamic equilibrium). For equilibrium to occur the resultant force must be zero (balanced forces) and the total moment must be zero.

Explosion: violent separation of two or more objects.

Force: the effect of a force will be to change the motion of an object.

Force, resultant: the combined effect of two or more forces.

Force, types of: two main types in mechanics, forces of attraction and contact forces.

Friction: the force which opposes motion when an object slides along a surface. Friction can prevent motion from happening provided the applied force does not exceed the maximum value for friction (μR).

Friction, coefficient of (μ): gives a measure of the roughness between two surfaces in contact.

Fulcrum: see Pivot.

Gravity: the force of attraction between two bodies due to their mass. This normally means the force of attraction between an object and the Earth. The force due to gravity is called weight.

Hinge: a fixed point about which a rigid body may turn.

Impulse–momentum principle:
Impulse = change in momentum
or $I = mv - mu$.

Inelastic: does not stretch.

Inextensible: does not stretch.

Impulse: is a force which acts on an object for a short time. The value of the impulse is given by Ft when the force is constant. Alternatively, the impulse is equal to the change in momentum of the object. The SI units of impulse are newton-seconds, N s.

Kinematics: the modelling of motion using displacements, velocities and accelerations (not forces).

Lamina: this is a thin plate or sheet of uniform thickness. Its mass is proportional to its area.

Light: having negligible or no mass.

Limiting equilibrium: occurs when the applied force on an object is equal to the maximum possible value of friction (μR). Any increase in the applied force will cause the object to accelerate.

Magnitude: is the length of a vector.

Mass: the measure of a body's tendency to oppose changes in its acceleration.

Modelling: a system where a mathematician works through a number of stages when solving a problem. The author of this book has chosen to use DMAI, D standing for define, M for model, A for analyse and I for interpret.

Moment: the measure of the turning effect of a force, it is the product of the force and the perpendicular distance to its line of action from the point.

Momentum: is the product of an object's mass and velocity. The units of momentum are N s or kg m s^{-1}.

Negative rotation: a clockwise rotation.

newton: the SI unit of force. Notice, like all units that are named after famous scientists, e.g. joule, the newton unit starts with a lower case 'n'. However, the abbreviation is upper case, i.e. 'N'.

Newton, Sir Isaac (1643–1727): UK mathematician, astronomer and physicist.

Newton's first law: a body will continue to remain at rest or move at constant speed in a straight line unless an external force makes it act otherwise.

Newton's second law: a resultant force acting on a body produces an acceleration which is proportional to the resultant force, *or*, the rate of change of linear momentum of a body is equal to the total applied force.

Newton's third law: for every action there is an equal and opposite reaction.

Normal reaction force: contact force that acts at right angles from the surface at the point of contact.

Particle model: this model ignores the size and shape of an object. The object's mass is treated as if it acts at a single point.

Pivot (a.k.a. fulcrum, or hinge): this is the point about which an object turns or rotates.

Positive rotation: an anticlockwise rotation.

Principle of conservation of linear momentum (CLM): states that the momentum during a collision remains constant as long as all motion remains in a straight line.

$$m_a u_A + m_b u_B = m_A v_A + m_B v_B$$

Proportion: for example, when resistance is proportional to speed, then $R \propto v$ giving $R = kv$ where k is the constant of proportion.

Radian: angular measure defined as the angle at the centre of a sector whose arc length is equal to the radius of the circle.

Resolving forces: separating a single force into two perpendicular components.

Resultant force: see Force, resultant.

Resultant vector: see Vector, resultant.

Resultant velocity: see Velocity, resultant.

Retardation: see Deceleration.

Rigid body: a simple shape with fixed lengths which do not change.

Rod: a simple model for a shape as a line with no thickness and a fixed length.

Rotation: a movement about an axis either in a positive or negative direction, where each point on the rotating body keeps a fixed distance from the axis (see positive and negative rotation).

Rough surfaces: two surfaces are said to be rough when a frictional force acts to oppose motion.

Scalar: a quantity which has size, but no direction.

Sine rule: a rule for calculating lengths and angles in triangles.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

where a, b and c are the lengths of the sides and A, B and C are the angles opposite these sides.

Smooth surface: surfaces are said to be smooth when the frictional force is negligible.

Speed, average: the distance travelled per unit of time:

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

Speed–time graph, area under: gives the distance travelled.

Speed–time graph, gradient: gives the acceleration.

Standard form (a.k.a. standard index form): a method of writing very large or very small numbers in the form of $a \times 10^n$, where $1 \leqslant a < 10$.

Static equilibrium: occurs when an object is stationary and remains so under the action of several forces.

Statics: the study of stationary equilibrium and structures in mechanics. Static means stationary, or not moving.

Strut: is a rod which is in compression. A strut exerts a thrust.

Surd: the square root of a number that cannot be written as a single fraction, e.g. $\sqrt{2}$.

Système International (SI) units: the standardised system of metric units, including metres, seconds, kg.

Tension: an internal force in a body which tries to prevent the body being stretched.

Thrust: an internal force in a body which tries to prevent the body being compressed.

Translation: a movement of an object where the characteristics of size and orientation of the object are maintained.

Triangle of forces: when three forces acting on a body in equilibrium are joined head to tail, they form a triangle. This triangle of forces can then be used to solve the problem using geometry.

Variation: see Proportion.

Vector: a quantity that has magnitude and direction, e.g. a speed of $60\,\text{km}\,\text{h}^{-1}$ on a bearing of 045°. Vectors obey the parallelogram rule of addition.

Vector, position: is the displacement of a body measured from a fixed origin, e.g. $\overrightarrow{OP}$ is the position vector of point P from a fixed point O.

Vector, resultant: the combined effect of two or more vectors.

Vector, unit: possesses a magnitude of one unit and can point in any direction. It is usual to denote the unit vector in the positive x-direction by **i**, the unit vector in the positive y-direction by **j** and the unit vector in the positive z-direction by **k**.

Vector, zero: this is the vector whose components are all zero. It is denoted as **0** or $\underline{0}$.

Velocity, average: ratio of distance travelled in a particular direction to the time taken for the journey.

Velocity, resultant: the combined effect of two or more velocities.

Velocity–time graph, area under: gives the displacement.

Weight: the force of attraction between the Earth and an object. The weight of an object equals mg, where m is the mass of the object in kg and g is the acceleration due to gravity, in $\text{m}\,\text{s}^{-2}$.

Revision Checklist

Unit	What I should know....	Book page	Date revised
Mathematical Models in Mechanics	• The common mathematical models; particle, rod, lamina, rigid body • The related common modelling attributes; light, uniform, non-uniform, smooth, inextensible • Other models; pulley, bead, peg, wire • That for a particle model, air resistance, rotation and length are ignored • That for a rigid body, lengths always remain constant		
Vectors in Mechanics	• That a vector has both magnitude and direction • That a scalar quantity has only magnitude • How to write a vector in component form using unit vectors (**i** and **j**), as a column vector or using end-point notation • How to find the magnitude and direction of a vector given in component form • How to resolve into perpendicular components a vector given as a length and a direction • How to add and subtract vectors and represent this on a vector diagram • That scalar multiples of a vector are parallel • How to use vectors to represent displacement, velocity, acceleration and various forces		
Kinematics	• How to draw the following graphs: distance–time, displacement–time, speed–time, velocity–time, acceleration–time • The meaning of the gradient on each of these types of graphs • The meaning of the area under each of these types of graphs • How to use dimensional analysis of the units to find the appropriate units for area or gradient on a graph • How to use the Uniform Acceleration Formulae, identifying which one to use for a given problem • How to use the Uniform Acceleration Formulae in vector form (avoiding using $v^2 = u^2 + 2as$) • That motion under gravity is a particularly common example of uniformly accelerated motion		
Forces	• The properties of a Force • How to represent forces using vectors • How to resolve forces into perpendicular components • How to add and subtract forces expressed in components • The definitions and applications of the following forces; weight, friction, tension, thrust, normal reaction • That $0 \leqslant F \leqslant \mu R$, where μ is the coefficient of friction • How to find the moment of a force about a point when the force is perpendicular to the distance and when it is at an angle to the distance • How to calculate the sum of the moments of several coplanar forces		